

## Processing one loop virtual corrections with SAMURAI

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work done in collaboration with P. Mastrolia, G. Ossola and T. Reiter

### OVERVIEW

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### Introduction

□ LHC successfully started collisions at 7 TeV on March 30<sup>th</sup> 2010

visit the LPCC web site for updates

http://lpcc.web.cern.ch/LPCC/

□ The need of Next to Leading Order (NLO) multi-particle scattering predictions is more pressing

New ideas in the field of loop corrections seems give the possibility to perform the automatic generation of NLO predictions for multi-leg processes

### Existing tools

### Leading Order

### NLO parton level

- ✓ MadGraph-MadEvent
- ✓ CompHep-CalcHep
- ✓ SHERPA
- ✓ WIZHARD
- ✓ ALPGEN
- ✓ HELAC
- ✓ .....

- ✓ MCFM
- ✓ NLOjet++
- ✓ .....
- NLO + parton shower
- ✓ MC@NLO
- ✓ POWHEG

#### General method for NLO parton integrator

□ The ingredients for a NLO prediction are:

- $\checkmark$  Tree graphs for the lowest order
- $\checkmark$  Tree graphs for the real radiation
- $\checkmark$  One loop correction to the Born level process

] The Born approximation involve m partons in the final state

$$\sigma^{LO} = \int_m d\sigma^B$$

□ At NLO we have the real cross section  $d\sigma^R$  with m+l partons in the final state and the one-loop correction  $d\sigma^V$  to the process with m partons in the final state

$$\sigma^{NLO} \equiv \int d\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$

] The two integrals are separately divergent although their sum is finite

#### Solution: subtraction method Ellis, Ross, Terrano (1981)

□ The general idea consists of the use of the identity

$$d\sigma^{NLO} = \left[ d\sigma^R - d\sigma^A \right] + d\sigma^A + d\sigma^V$$

□ Where  $d\sigma^A$  is a proper approximation of  $d\sigma^R$  such as to have the same singular behavior point-by-point as  $d\sigma^R$  itself.

$$\sigma^{NLO} = \int_{m+1} \left[ d\sigma^R - d\sigma^A \right] + \int_{m+1} d\sigma^A + \int_m d\sigma^V$$

□ Further  $d\sigma^A$  can be chosen in such a way to be analytically integrable over the extra parton degrees of freedom. Adding it back to the virtual correction we form a finite m parton integrand

$$\sigma^{NLO} = \int_{m+1} \left[ d\sigma^R - d\sigma^A \right] + \int_m \left[ d\sigma^A + d\sigma^V \right]$$

# Status of the art

Analytic calculations;

 $\Box$  W/Z/y+ 2jets Bern et al (1998)

□ H + 2jets (Badger, Berger, Campbell, Del Duca, Dixon, Ellis, Glover, Mastrolia, Risager, Sofianatos, Williams) (2006-2009) Numerical calculations;

- □ EW corr. e+e- > 4 fermions Denner and Dittmaier (2005)
- p > W + 3jets Ellis et al, Berger et al (2009)
- □ pp > Z + 3jets Berger et al (2009)
- □ pp > ttbb Bredenstein et al, Bevilacqua et al (2009)
- p > tt +2jets Czakon et al
   (2010)
- □ pp > 4b Binoth et al (2010)

### Methods

Basic features of SAMURAI Scatterig AMplitudes from Unitarity based Reduction Algorithm at Integrand level Authors: P. Mastrolia, G. Ossola, T. Reiter and F.T.

- Is a fortran90 library for the calculation of the virtual corrections downloadable at the URL: <u>www.cern.ch/samurai</u>
- Main purpose was to provide a flexible and easy to use tool for the evaluation of the virtual corrections
- It works with any number/kind of legs
- Can process integrands written either as numerator of Feynman diagrams or as product of tree level amplitudes
- Can be compiled in double or quadruple precision
- Many details including examples of applications can be found in arXiv:1006.0710 NAPOLI 10/06/2010

#### OPP reduction algorithm 0. the idea

□ Any amplitude can be expressed as a linear combination of scalar integrals: boxes, triangles, bubbles, tadpoles plus rational terms

$$\int A = \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3) + \sum_{i_0 < i_1 < i_2}^{m-1} c(i_0 i_1 i_2) C_0(i_0 i_1 i_2) + \sum_{i_0 < i_1}^{m-1} b(i_0 i_1) B_0(i_0 i_1) + \sum_{i_0}^{m-1} a(i_0) A_0(i_0) + \text{ rational terms}$$

At integrand level the structure is enriched by terms that integrate to zero

$$N(q) = \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[ d(i_0i_1i_2i_3) + \tilde{d}(q;i_0i_1i_2i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0 < i_1 < i_2}^{m-1} \left[ c(i_0i_1i_2) + \tilde{c}(q;i_0i_1i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i + \sum_{i_0 < i_1 < i_2}^{m-1} \left[ b(i_0i_1) + \tilde{b}(q;i_0i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[ a(i_0) + \tilde{a}(q;i_0) \right] \prod_{i \neq i_0}^{m-1} D_i$$

#### OPP reduction algorithm 1. the idea

□ Once fixed a parametrization for the loop momentum in terms of a linear combination of known four-vectors the vanishing term are polynomial

 $q = -p_0 + x_1e_1 + x_2e_2 + x_3e_3 + x_4e_4$ 

For example the box residue reads:

$$\Delta_{ijk\ell}(\bar{q}) = c_{4,0}^{(ijk\ell)} + c_{4,2}^{(ijk\ell)} \mu^2 + c_{4,4}^{(ijk\ell)} \mu^4 - \left(c_{4,1}^{(ijk\ell)} + c_{4,3}^{(ijk\ell)} \mu^2\right) \left[ (K_3 \cdot e_4) x_4 - (K_3 \cdot e_3) x_3 \right] (e_1 \cdot e_2)$$

☐ The problem is then reduced to fit the coefficients of the polynomials

$$N(\bar{q}) = \sum_{i < < m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h \neq i,j,k,\ell,m}^{n-1} \bar{D}_h + \sum_{i < \ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h \neq i,j,k,\ell}^{n-1} \bar{D}_h + \sum_{i < j}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h \neq i,j,k}^{n-1} \bar{D}_h + \sum_{i < j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h \neq i,j}^{n-1} \bar{D}_h + \sum_{i < j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h \neq i,j}^{n-1} \bar{D}_h + \sum_{i < j}^{n-1} \Delta_{ii}(\bar{q}) \prod_{h \neq i}^{n-1} \bar{D}_h$$

### OPP reduction algorithm 2. generalized cuts

With appropriate parametrizations one can strongly simplify the problem of fitting the coefficient of the polynomials -> cuts construction -> recursive solution (top-down)

Choosing the loop momentum q such that a set of denominators vanish leads to a triangular solutions for the system of the coefficients…

### d-dimensional generalized unitarity cuts

The polynomials can encode also the mu2 dependence giving rise to the rational part Giele, Kunszt, Melnikov (2008); Ellis, Giele, Kunszt, Melnikov (2008)

 $\Delta_{ijk\ell}(\bar{q}) = c_{4,0}^{(ijk\ell)} + c_{4,2}^{(ijk\ell)} \mu^2 + c_{4,4}^{(ijk\ell)} \mu^4 - \left(c_{4,1}^{(ijk\ell)} + c_{4,3}^{(ijk\ell)} \mu^2\right) \left[ (K_3 \cdot e_4) x_4 - (K_3 \cdot e_3) x_3 \right] (e_1 \cdot e_2)$ 

### An implementation of the D-dimensional generalized unitarity cuts technique

The power of the method is the fact that for each phase space point the only info required to perform the reduction is the knowledge of the numerical value of the numerator  $N(q, mu2, \epsilon)$ for a finite set of values for the loop momentum (q,mu2)

### Discrete Fourier Transform

1. generate the set of discrete values  $P_k$  (k = 0, ..., n),

$$P_k = P(x_k) = \sum_{\ell=0}^n c_\ell \ \rho^\ell \ e^{-2\pi i \frac{k}{(n+1)}\ell}$$

by sampling P(x) at the points

$$x_k = \rho \ e^{-2\pi i \frac{k}{(n+1)}}$$

2. using the orthogonality relation



$$\sum_{n=0}^{N-1} e^{2\pi i \frac{k}{N} n} e^{-2\pi i \frac{k'}{N} n} = N \,\delta_{kk'}$$

each coefficient  $c_{\ell}$  finally reads,

$$c_{\ell} = \frac{\rho^{-\ell}}{n+1} \sum_{k=0}^{n} P_k e^{2\pi i \frac{k}{(n+1)}\ell}$$

The extension of the DFT projection to the case of multi-variate polynomials is straightforward

### Amplitudes & Master Integrals

$$\begin{aligned} \mathcal{A}_{n} &= \sum_{i < j < k < \ell}^{n-1} \left\{ c_{4,0}^{(ijk\ell)} I_{ijk\ell}^{(d)} + \frac{(d-2)(d-4)}{4} c_{4,4}^{(ijk\ell)} I_{ijk\ell}^{(d+4)} \right\} & \int d^{d}\bar{q} \frac{\bar{q} \cdot e_{2}}{\bar{D}_{i}\bar{D}_{j}} = J_{ij}^{(d)} \\ &+ \sum_{i < j < k}^{n-1} \left\{ c_{3,0}^{(ijk)} I_{ijk}^{(d)} - \frac{(d-4)}{2} c_{3,7}^{(ijk)} I_{ijk}^{(d+2)} \right\} & \int d^{d}\bar{q} \frac{(\bar{q} \cdot e_{2})^{2}}{\bar{D}_{i}\bar{D}_{j}} = K_{ij}^{(d)} \\ &+ \sum_{i < j}^{n-1} \left\{ c_{2,0}^{(ij)} I_{ij}^{(d)} + c_{2,1}^{(ij)} J_{ij}^{(d)} + c_{2,2}^{(ij)} K_{ij}^{(d)} - \frac{(d-4)}{2} c_{2,9}^{(ij)} I_{ij}^{(d+2)} \right\} \\ &+ \sum_{i < j}^{n-1} c_{1,0}^{(i)} I_{i}^{(d)} \end{aligned}$$

Sources of rational terms are the integrals with mu2 powers in the numerator

They are generated by the reduction algorithm, but could also be present ab initio in the numerator function as a consequence of the algebraic manipulations

$$\int d^{d}\bar{q} \frac{\mu^{2}}{\bar{D}_{i}\bar{D}_{j}} = -\frac{(d-4)}{2} I_{ij}^{(d+2)}$$

$$\int d^{d}\bar{q} \frac{\mu^{4}}{\bar{D}_{i}\bar{D}_{j}\bar{D}_{k}\bar{D}_{\ell}} = \frac{(d-2)(d-4)}{4} I_{ijk\ell}^{(d+4)}$$

$$\int d^{d}\bar{q} \frac{\mu^{2}}{\bar{D}_{i}\bar{D}_{j}\bar{D}_{k}} = -\frac{(d-4)}{2} I_{ijk}^{(d+2)}$$

### Running SAMURAI

### calls:

- call initsamurai(imeth, isca, verbosity, itest)
- call InitDenominators(nleg,Pi,msq,v0,m0,v1,m1,...,vlast,mlast)

call samurai(xnum,tot,totr,Pi,msq,nleg,rank,istop,scale2,ok)
call exitsamurai

A dedicated module (kinematic) is also available in the release that contains useful functions to evaluate:

- ✓ Polarization vectors for massless vectors
- ✓ Scalar and spinor products with both real and complex four vectors as arguments

#### call initsamurai(imeth, isca, verbosity, itest)

- ✓ imeth = 'diag' for an integrand given as numerator of a Feynman diagram 'tree' for an integrand given as the product of tree level amplitudes
- ✓ isca = 1, scalar integrals evaluated with the QCDLoop package (Ellis and Zanderighi)
   2, scalar integrals evaluated with the AVH-OLO package (van Hameren)
- $\checkmark$  verbosity = 0, nothing is printed by the reduction
  - 1, the coefficients are printed out
  - 2, also the value of the MI are printed out
  - 3, also the results of the tests are printed out
- $\checkmark$  itest = 0, none test
  - 1, global n=n test is performed (not avail. for imeth= 'tree')
  - 2, local n=n test is performed
  - 3, power test is performed (not avail. for imeth= 'tree') new - based on the mismatch of the polynomial degree of the given integrand and the reconstructed one

#### Optionally to fill the denominators one can use

call InitDenominators(nleg,Pi,msq,v0,m0,v1,m1,...,vlast,mlast)



Denominator(j) =  $[q + Pi(j,:)]^2 - mu2 - msq(j)$ 

call samurai(xnum,tot,totr,Pi,msq,nleg,rank,istop,scale2,ok)

- xnum [i]= the name of the function to reduce with arguments xnum(cut, q, mu2)
  for imeth=tree the cut play a selective role to use the relative
  tree product
- $\checkmark$  tot [o] = contains the result of the reduction convoluted with the MI
- $\checkmark$  totr [o]= contains the rational part only
- $\checkmark$  rank [i] = the rank of the numerator, useful to speed up the reduction
- $\checkmark$  istop [i] = when stop the reduction, i.e. after pentuple cut (5) quadruple (4)...
- $\checkmark$  scale2 [i] = the value of the renormalization scale (square)
- $\checkmark$  ok [o] = a logical variable giving the result of the test if they are evaluated

### About the precision

- ✓ Gram Determinant -> induce large cancellations between contributions from the MI that carry such a factor (the tests coded in SAMURAI detect such instabilities)
- $\checkmark$  Big cancellations between diagrams -> on-shell methods seems to be the best option
- ✓ If running with big internal masses -> big cancellations between cut-constructible and rational term -> effective theory works better

Quadruple precision solves these issues, but is time consuming For numerical studies and checks SAMURAI compiles also in quad

### Examples

4-photons



- imeth= 'diag'
- nleg = 4, rank = 4
- 6 permutations, only 3 relevant

$$\begin{split} \bar{L}_1 &= \bar{q} \ , \ \bar{L}_2 = \bar{q} + p_2 \ , \ \bar{L}_3 = \bar{q} + p_{23} \ , \ \bar{L}_4 = \bar{q} + p_{234} \\ N(\bar{q}) &= -\text{Tr}\Big[ (\bar{L}_1 + m) \not e_2 \ (\bar{L}_2 + m) \not e_3 \ (\bar{L}_3 + m) \not e_4 \ (\bar{L}_4 + m) \not e_1 \Big] \end{split}$$

- mu2 terms give zero contribution
- mu2 q^al q^be cancel in the sum
- mu2^2 gives rise to the correct rational part

Results numerically checked vs. Gounaris et al (1999)

### 6-photons



PS point as in Nagy and Soper (2006)  $\vec{p}_3 = (33.5, 15.9, 25.0)$   $\vec{p}_4 = (-12.5, 15.3, 0.3)$   $\vec{p}_5 = (-10.0, -18.0, -3.3)$  $\vec{p}_6 = (-11.0, -13.2, -22.0)$ 

- imeth = 'diag'
- nleg = 6, rank = 6
- 120 permutations, only 60 relevant

 $\begin{array}{l} \mbox{Bernicot et al (2007,2008)} \\ \frac{s}{\alpha^3} \, A(-,-,+,+,+,+) \, = \, 11075.04009210435 \ , \\ \frac{s}{\alpha^3} \, A(+,-,-,+,+,-) \, = \, 7814.762085902767 \ , \end{array}$ 

 $\begin{array}{l} \mbox{SAMURAI with istop=2} \\ \frac{s}{\alpha^3}\,A(-,-,+,+,+,+) = \underline{11075.040}174990 \ , \\ \frac{s}{\alpha^3}\,A(+,-,-,+,+,-) = \underline{7814.762}3429908 \ . \end{array}$ 

SAMURAI with istop=4, subtracting totr $\frac{s}{\alpha^3} A(-, -, +, +, +, +) = \underline{11075.040092102} ,$  $\frac{s}{\alpha^3} A(+, -, -, +, +, -) = \underline{7814.76208590} 84 .$ 

Results numerically checked vs. Bernicot et al (2007,2008)

### 8-photons



- imeth = 'diag'
- nleg = 8, rank = 8
- 5040 permutations, only 2520 relevant
- sampling set as in Gong et al (2008)

MHV result numerically checked vs. Mahlon (1993)

NNMHV result (new) numerically confirm the structure in Badger et al (2009) The points in quadruple precision (x) have been calculated with istop=2, i.e. retaining all the cut constructble and rational pieces



### Drell-Yan



If one want to consider regularization schemes giving rise to  $O(\epsilon)$  terms and reduce them, then one needs to process  $N_0$  and  $N_1$  below separately

$$\mathcal{N}(\bar{q},\epsilon) = N_0(\bar{q}) + \epsilon N_1(\bar{q}) + \epsilon^2 N_2(\bar{q}).$$

d=4 -> Dim Red d=4-2ε -> CDR

- imeth = 'diag'
- nleg = 3, rank = 2

**Denominators:**  $\bar{q}^2 (\bar{q} + p_u)^2 (\bar{q} + p_u + p_{e^-} + p_{e^+})^2$ 

•  $msq = \{ 0, 0, 0 \}$ 

•  $Pi = \{ \underline{0}, p_u, p_u + p_{e^-} + p_{e^+} \}$ 

•  $N_1$  generate a rational term = -  $g_s^2 C_F LO$ 

VB+lj: *leading color* 



- imeth = 'diag'
- l Box nleg=4, rank=3
  4 Tri nleg=3, rank=2
  2 Bub nleg=2, rank=1
- Diagrams can be collected on a common box denominator
- Studing Left-handed current needs of a prescription for gamma5: adopting DR w/anticommuting gamma5 we added  $-N_c/2$  times the Tree Level amplitude

Results numerically checked vs. Bern et al (1997) Eqs D1-5, using some code from MCFM







#### 6-gluons all plus: massive scalar contribution



- imeth= 'tree'
- nleg = 6, rank = 6

$$\begin{split} A_3^{\text{tree}}(1_s; 2^+; 3_s) &= \frac{[2|1|r_2\rangle}{\langle 2 \, r_2 \rangle} \ , \\ A_4^{\text{tree}}(1_s; 2^+, 3^+; 4_s) &= \frac{\mu^2 \ [2 \, 3]}{\langle 2 \, 3 \rangle (p_{12}^2 - \mu^2)} \ , \\ A_5^{\text{tree}}(1_s; 2^+, 3^+, 4^+; 5_s) &= \frac{\mu^2 \ [2|1 \ (2+3)|4]}{\langle 2 \, 3 \rangle \langle 3 \, 4 \rangle (p_{12}^2 - \mu^2) (p_{45}^2 - \mu^2)} \ , \end{split}$$

$$\begin{split} N(q,\mu^2) = & A_4(L_1;1^+,2^+;-L_2) \times A_3(L_2;3^+;-L_3) \times A_3(L_3;4^+;-L_4) \\ \times & A_3(L_4;5^+;-L_5) \times A_3(L_5;6^+;-L_1) \end{split}$$

For this helicity choice the result is purely rational

Results numerically checked vs. Badger et al (2005)



### 6q amplitudes 0. calculation



Fortran Code generation completely automated thanks to an infrastructure derived from Golem-2.0

### 6q amplitudes .1 checks

- A(-+-+-+)
- ren scale = 1GeV
- uv renormalization included

 $\begin{aligned} \vec{p}_3 &= (33.5, 15.9, 25.0) \\ \vec{p}_4 &= (-12.5, 15.3, 0.3) \\ \vec{p}_5 &= (-10.0, -18.0, -3.3) \\ \vec{p}_6 &= (-11.0, -13.2, -22.0) \end{aligned} \qquad a_{\rm LO} = \mathcal{A}_{\rm LO}^{\dagger} \mathcal{A}_{\rm LO} \\ a_{\rm LO} &= \mathcal{A}_{\rm LO}^{\dagger} \mathcal{A}_{\rm LO} \\ \vec{p}_6 &= (-11.0, -13.2, -22.0) \end{aligned}$ 

GOLEM-2.0 + GOLEM95

 $a_{\rm LO} = 0.9686295685264447 \times 10^{-6}$ ,  $a_{-2} = -8.00000000048633$ ,  $a_{-1} = 46.40675046335535$ ,  $a_0 = -233.8908276457752$ ;

#### GOLEM-2.0 + SAMURAI

 $a_{\rm LO} = 0.9686295685264458 \times 10^{-6}$ ,

 $a_{-2} = -7.99999999999935$  ,

 $a_{-1} = 46.40675045992446$  ,

 $a_0 = -233.8908276128404$ ,

### 6q amplitudes .2 precision



Difference between the single (double) virtual poles and those of the integrated dipoles for 10^5 phase space points

### Conclusions

- We wrote the SAMURAI library for the automatic evaluation of the NLO virtual correction to scattering processes, once the integrand is given in some form: Feynman diagrams or product of tree level amplitudes
- I showed its main features and several examples that could be useful to understand the framework and as a guide to implement other processes
- We tried to make things as effective and simple as possible to allow for interfaces with other tools

### Outlook

- Improve on velocity and stability Especially for Degenerate kinematic configurations
- In the near future we plan to study some processes relevant for Higgs particle discovery at the LHC:
  - $\checkmark$  H production in association with 3jets

and important background processes for H and BSM searches at the LHC like:

- $\checkmark$  4-top production
- ✓ WW+2j production