# Equilibrium Statistical Mechanics of Spin and Fermion Lattice Systems

## - New technique and results -

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The aim of this talk is to present an introductory account of recent works of myself and Moriya, which give vast improvements over those in the book of Bratteli and Robinson about the equivalence of different characterizations of equilibrium states such as the KMS condition and the variational principle, by a use of a non-commutative conditional expectation. (A detailed account of this theory will be given in the series of lectures at the Department of Mathematics of University of Rome 1.)

We consider a lattice system with a finite number of spins and Fermions at each lattice point. Due to the presence of Fermions, the algebras of physical quantities at different lattice points do not commute, a situation which has not necessarily been considered seriously up to now (except for a few papers). With the motivation of treating such situations, we introduce a conditional expectation from the whole algebra onto each local subalgebra localized on a subset of the lattice, which is defined in reference to a product state (such as the tracial state or the Fermion vacuum state). This turns out to be very effective.

On one hand, for every dynamics which is even with respect to the Fermion number, (more precisely, for every even * derivation defined on the union of all finitely localized subalgebras of physical quantities, independent of whether the derivation is a generator of a dynamics or not), we can associate a unique standard potential, which defines a local Hamiltonian operator and hence the given * derivation as its commutator as usual. This potential automatically satisfies a cetain convergence condition which we have not imposed arbitrarily but comes out from the definition of the potential from the given * derivation. The unique correspondense of the potential for a given * derivation (the generator of the dynamics) is obtained by the standardness condition for the potential, expressed as vanishing of some of its conditional expectations.

Under this general framework, without any further assumptions on the potential, we can obtain equivalence of various characterizations of equilibrium states. This is a vast improvement over older results, for which one needs some strong convergence condition about potentials.

**Reference**

[1] H.Araki and H.Moriya, Equilibrium statistical mechanics of Fermion lattice systems, Rev.Math.Phys. **15** (2003) 93-198.

[2] H.Araki and H.Moriya, Local thermodynamical stability of Fermion lattice systems, Lett.Math.Phys. **62** (2003) 33-45.

[3] O.Bratteli and D.W.Robinson, Operator Algebras and Quantum Statistical Mechanics, vol.2, second ed. (Springer Verlag, 1997).