The Strategy Logic Saga

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1 Extended Abstract

In open-system verification, a fundamental area of research is the study of modal logics for strategic reasoning in the setting of multi-agent games [3, 5]. An important contribution in this field has been the development of *Alternating-Time Temporal Logic* (ATL*, for short), introduced by Alur, Henzinger, and Kupferman [1]. Formally, it is obtained as a generalization of the logic CTL* [4], where the path quantifiers *there exists* "E" and *for all* "A" are replaced with strategic modalities of the form "$\langle\langle A\rangle\rangle$" and "$[[A]]$", for a set A of *agents*. These modalities are used to express cooperation and competition among agents in order to achieve a temporal goal. Several decision problems have been investigated about ATL*; both its model-checking and satisfiability problems are decidable in 2EXPTIME [10], just like it is for CTL*.

Despite its powerful expressiveness, ATL* suffers from the strong limitation that strategies are treated only implicitly through modalities that refer to games between competing coalitions. To overcome this problem, Chatterjee, Henzinger, and Piterman introduced *Strategy Logic* (CHP-SL, for short) [2],which treats strategies in *two-player turn-based games* as *first-order objects*. The explicit treatment of strategies makes this logic very useful and more expressive than ATL*, however, it still suffers from severe limitations. In particular, it is limited to two-player turn-based games and does not allow different players to share the same strategy, suggesting that strategies have yet to become truly first-class objects in this logic. For example, it is impossible to describe the classic strategy-stealing argument of many real-life combinatorial games.

These considerations has led us to introduce and investigate a new *Strategy Logic*, denoted SL, as a more general framework than CHP-SL, for explicit reasoning about strategies in multi-agent concurrent games [8]. Syntactically, SL extends the logic LTL [9] by means of *strategy quantifiers*, the existential $\langle\langle x\rangle\rangle$ and the universal $[[x]]$, as well as *agent binding* (a, x), where a is an agent and x a variable. Intuitively, these elements can be read as *"there exists a strategy x"*, *"for all strategies x"*, and *"bind agent a to the strategy associated with x"*, respectively.

The price that one has to pay for the expressiveness of SL w.r.t. ATL* is the lack of important model-theoretic properties and an increased complexity of related decision problems. In particular, in [6, 8], it was shown that SL does not have the bounded-tree model property and the satisfiability problem is *highly undecidable*, precisely, Σ_1^1-HARD. Moreover, in [7], it was shown that the model checking problem is nonelementary-complete (we recall that also for CHP-SL it is known to be nonelementary, while it is open the question whether it is decidable).

The negative complexity results on the decision problems of SL with respect ATL*, provide motivations for an investigation of decidable fragments of SL, strictly subsuming ATL*, with a better complexity. In particular, by means of these sublogics, one may understand why SL is computationally more difficult than ATL*.

The main fragments we have investigated and studied are *Nested-Goal*, *Boolean-Goal*, and *One-Goal Strategy Logic*, respectively denoted by SL[NG], SL[BG], and SL[1G]. They encompass formulas in a special prenex normal form having nested temporal goals, Boolean combinations of goals, and a single goal at a time, respectively. For goal we mean an SL formula of the type $\flat\psi$, where $\flat$ is a binding prefix of the form $(\alpha_1, x_1), \ldots, (\alpha_n, x_n)$ containing all the involved agents and ψ is an agent-full formula. In SL[1G], each temporal formula ψ is prefixed by a quantification-binding prefix $\wp\flat$ that quantifies over a tuple of strategies and binds them to all agents.

As main results about these fragments, we have prove that the satisfiability and model-checking problems for

SL[1G] are 2EXPTIME-COMPLETE, thus not harder than the one for ATL*. On the contrary, for SL[NG], the model checking problem is nonelementary and the satisfiability is undecidable. Finally, we observe that SL[BG] includes CHP-SL, the relative model-checking problem relies between 2EXPTIME and NONELEMENTARYTIME, while the satisfiability problem is undecidable.

To achieve all positive results about SL[1G], we use a fundamental property of the semantics of this logic, called *elementariness*, which allows us to strongly simplify the reasoning about strategies by reducing it to a set of reasonings about actions. This intrinsic characteristic of SL[1G], which unfortunately is not shared by the other fragments, asserts that, in a determined history of the play, the value of an existential quantified strategy depends only on the values of strategies, from which the first depends, on the same history. This means that, to choose an existential strategy, we do not need to know the entire structure of universal strategies, as for SL, but only their values on the histories of interest.

By means of elementariness, we can solve the SL model-checking procedure via alternating tree automata in such a way that we avoid the projection operations by using a dedicated automaton that makes an action quantification for each node of the tree model. As this automaton is only exponential in the size of the formula (and independent from its alternation number) and its nonemptiness can be computed in exponential time, we get that the model-checking procedure for SL[1G] is 2EXPTIME. Clearly, the elementariness property also holds for ATL*, as it is included in SL[1G]. In particular, although it has not been explicitly stated, this property is crucial for most of the results achieved in literature about ATL* by means of automata.

Out of this picture, SL[1G] is the biggest known decidable fragment of SL, strictly subsuming ATL*. One the other side, the bigger but undecidable logic SL[BG] is of major interest. Indeed, it can describe several interesting system properties non expressible in SL[1G] such as *nash equilibrium, strong nash equilibrium, sub-game perfect equilibrium, coalition proof nash equilibrium*, etc. This reason, along with the open model-checking question, has spurred us to keep investigating SL[BG] as well as some of its proper fragments strictly subsuming SL[1G]. Among the other, we have prooved that if we restrict the logic to only consider conjunctive combinations of goals (SL[CG]), the model checking problem is solvable in 2EXPTIME, while it remains open its decidability question. The model checking positive result is obtained by recovering a sort of more general elementariness for this logic, which now is related to goals more than single configurations of the system. In practice, this elementariness ensures the maintaince of the coherence among the conjucntive goals, even when they are treated singularly.

All the results reported in this paper, but those related to SL[CG], come from [6, 7, 8]. The interested reader can refer to these works to find more motivations, examples and related material.

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