# Strategy Logic Fragments

Fabio Mogavero, Aniello Murano and Giuseppe Perelli
*Università degli Studi di Napoli "Federico II", Napoli, Italy.*
Moshe Y. Vardi
*Rice University, Houston, Texas, USA.*

## 1 Extended Abstract

Strategy Logic (SL, for short) has been recently introduced by Mogavero, Murano, and Vardi as a formalism for reasoning explicitly about strategies, as first-order objects, in multi-agent concurrent games [3]. This logic turns to be very powerful, strictly subsuming all major previously studied modal logics for strategic reasoning, including ATL, ATL*, and the like. The price that one has to pay for the expressiveness of SL w.r.t. ATL* is the lack of important model-theoretic properties and an increased complexity of related decision problems. In particular, in [1, 3], it was shown that SL does not have the bounded-tree model property and the satisfiability problem is *highly undecidable*, precisely, $\Sigma_1^1$-HARD. Moreover, in [2], it was shown that the model checking problem is nonelementary-complete (we recall that also for CHP-SL it is known to be nonelementary, while it is open the question whether the related satisfiability problem is decidable or not).

The negative complexity results on the decision problems of SL with respect ATL*, provide motivations for an investigation of decidable fragments of SL, strictly subsuming ATL*, with a better complexity. In particular, by means of these sublogics, one may understand why SL is computationally more difficult than ATL*.

The main fragments we have introduced and studied are *Nested-Goal*, *Boolean-Goal*, and *One-Goal Strategy Logic*, respectively denoted by SL[NG], SL[BG], and SL[1G]. They encompass formulas in a special prenex normal form having nested temporal goals, Boolean combinations of goals, and a single goal at a time, respectively. For goal we mean an SL formula of the type $\flat\psi$, where $\flat$ is a binding prefix of the form $(\alpha_1, x_1), \ldots, (\alpha_n, x_n)$ containing all the involved agents and $\psi$ is an agent-full formula. In SL[1G], each temporal formula $\psi$ is prefixed by a quantification-binding prefix $\wp\flat$ that quantifies over a tuple of strategies and binds them to all agents.

As main results about these fragments, we have proved that the satisfiability and model-checking problems for SL[1G] are 2EXPTIME-COMPLETE, thus not harder than the one for ATL*. On the contrary, for SL[NG], the model checking problem is nonelementary and the satisfiability is undecidable. Finally, for SL[BG] the relative model-checking problem relies between 2EXPTIME and NONELEMENTARYTIME, while the satisfiability problem is undecidable.

To achieve all positive results about SL[1G], we use a fundamental property of the semantics of this logic, called *elementariness*, which allows us to strongly simplify the reasoning about strategies by reducing it to a step-by-step reasoning about which action to perform. This intrinsic characteristic of SL[1G], which unfortunately is not shared by the other two fragments, asserts that, in a determined history of the play, the value of an existential quantified strategy depends only on the values of strategies, from which the first depends, on the same history. This means that, to choose an existential strategy, we do not need to know the entire structure of universal strategies, as for SL, but only their values on the histories of interest.

By means of elementariness, we can solve the SL[1G] decision problems via alternating tree automata in such a way that we avoid the projection operations by using a dedicated automaton that makes an action quantification for each node of the tree model. As this automaton is only exponential in the size of the formula (and independent from its alternation number) and its nonemptiness can be computed in exponential time, we get that both model-checking and satisfiability for SL[1G] are 2EXPTIME. Clearly, the elementariness property also holds for ATL*, as it is included in SL[1G]. In particular, although it has not been explicitly stated, this property is crucial for most of the results achieved in literature about ATL* by means of automata.

All the results reported here come from [1, 2, 3]. The interested reader can refer to these works to find more motivations, examples and related material.

## References


[1] F. Mogavero, A. Murano, G. Perelli, and M. Y. Vardi. Reasoning about strategies: On the model-checking problem. Technical Report 1112.6275, arXiv, December 2011.

[2] F. Mogavero, A. Murano, G. Perelli, and M.Y. Vardi. A Decidable Fragment of Strategy Logic. Technical Report 1202.1309, arXiv, February 2012.

[3] F. Mogavero, A. Murano, and M.Y. Vardi. Reasoning About Strategies. In *IARCS Annual Conf. on Foundations of Software Technology and Theoretical Comp. Science'10*, LIPIcs 8, pages 133–144, 2010.