VIRTUAL STUDIES IN GRATING-BASED PHASE-CONTRAST IMAGING

Janne Vignero
INTRODUCTION

Talbot-Lau Interferometry (TLI)

Transmission Image  Differential phase Image  Dark Field Image
INTRODUCTION

TLI for mammography

Transmission Image

Differential phase Image

Soft tissue contrast

Calcifications

Dark Field Image
INTRODUCTION

TLI for mammography

Transmission Image

Dark Field Image

Calcifications

Comparison via contrast-to-noise ratios
TLI for mammography

**INTRODUCTION**

Transmission Image  
Differential phase Image

Soft tissue contrast

Comparison via contrast-to-noise ratios

**Simulations**  
**Detectability Study**  
**Applications**  
**Conclusion**
How to quantitatively compare Tr and dP imaging?
OUTLINE

- Talbot-Lau interferometry
- A hybrid simulation framework
  - generate ‘realistic’ imagines that match those
    of a TLI scanner
- A detectability study
  - a task-based study
  - human reader studies (4-AFC)
- Application: mammography
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- Talbot-Lau interferometry
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- Application: mammography
TALBOT-LAU INTERFEROMETRY (TLI)

ILLUSTRATION

Illumination by a **homogeneous** x-ray field

Creates intensity disturbances at the edges
TALBOT-LAU INTERFEROMETRY (TLI)

Illumination by a **homogeneous** x-ray field

- Creates intensity disturbances at the edges

Illumination by a **periodic** x-ray field

- Allows to measure the intensity shifts in addition to the edges
Also referred to as ‘grating-based’ phase-contrast imaging

Periodic x-ray field is created by a grating; ‘the Talbot effect’
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Periodic x-ray field is *created* by a grating; ‘the Talbot effect’

Periodic x-ray field is *measured* by a grating

For each pixel we measure an average intensity pattern
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Conventional x-ray tubes are not coherent
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INTRODUCTION

TLI SIMULATIONS

DETECTABILITY STUDY

APPLICATIONS

CONCLUSION
For each pixel we measure an average intensity pattern with and without object
For each pixel we measure 3 parameters → 3 images can be constructed.
TALBOT-LAU INTERFEROMETRY (TLI)

Transmission Image

Differential phase Image

\[ S_{Tr} = \exp(-\mu t) \]
\[ = \exp(-2k\beta t) \]

\[ S_{dp} = \frac{2\pi d}{p_2} \tan \left( \frac{\delta \delta t}{\delta x} \right) \]
RESEARCH QUESTION

How to quantitatively compare Tr and dP imaging?

Transmission Image

Differential phase Image
Transmission Versus Differential Phase Imaging

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| **Noise**         |                         |
| $\sigma_{\text{Tr}} \propto \frac{1}{\sqrt{PV}}$ | $\sigma_{\text{dP}} \propto \frac{1}{\sqrt{PV}} \cdot \frac{1}{v}$ |
## Transmission versus Differential Phase Imaging

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| **Noise**         | **Noise**               |
| $\sigma_{Tr} \propto \frac{1}{\sqrt{PV}}$   | $\sigma_{dp} \propto \frac{1}{\sqrt{PV}} \cdot \frac{1}{v}$ |

### 1. Beta versus delta
1. BETA VERSUS DELTA

For soft tissues $\delta \approx 1000 \cdot \beta$

$\neq$ 1000 times better performance of dP in comparison to Tr
**Transmission Versus Differential Phase Imaging**

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1. Beta versus delta
### TRANSMISSION VERSUS DIFFERENTIAL PHASE IMAGING

#### Signal

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#### Noise

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1. Beta versus delta

2. ‘\( d, p_2 \)’ the sensitivity

---

#### INTRODUCTION

- TLI SIMULATIONS
- DETECTABILITY STUDY
- APPLICATIONS
- CONCLUSION
3. THE SYSTEM SENSITIVITY

\[ S_{Tr} = \exp(-\mu t) = \exp(-2k\beta t) \]

\[ S_{dp} = \frac{2\pi d}{p_2} \tan \left( \frac{\partial \delta t}{\partial x} \right) \]
3. THE SYSTEM SENSITIVITY

The G1-to-G2 distance ‘d’

\[ \frac{2\pi d}{p_2} \]
3. THE SYSTEM SENSITIVITY

The G1-to-G2 distance ‘d’

\[ \frac{2\pi d}{p_2} \]

The system sensitivity
3. THE SYSTEM SENSITIVITY

The G1-to-G2 distance ‘d’

The system sensitivity

\[ \frac{2\pi d}{p_2} \]

The period of the interference pattern ‘p_2’
### Transmission Versus Differential Phase Imaging

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| **Noise**        |                         |
| $\sigma_{Tr} \propto \frac{1}{\sqrt{PV}}$       | $\sigma_{dp} \propto \frac{1}{\sqrt{PV}} \cdot \frac{1}{v}$ |

1. Beta versus delta
2. ‘$d, p_2$’ the system sensitivity
3. ‘$v$’, the system visibility
3. THE SYSTEM VISIBILITY

The visibility

\[ v = \frac{a_1}{a_0} \]

Decreased by

- Polychromatic source
- Finite width G0 slits
- Finite height G2 grating
- Beam divergence
The visibility

\[ v = \frac{a_1}{a_0} \]

Determines noise in dP image

\[ \begin{align*}
    v &= 0.9 \\
    v &= 0.6 \\
    v &= 0.4 \\
    v &= 0.25
\end{align*} \]
3. THE SYSTEM VISIBILITY

The visibility

\[ v = \frac{a_1}{a_0} \]

Determines noise in dP image

- \( v = 0.9 \)
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The visibility

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\[ v = 0.9 \quad v = 0.6 \]

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Determines noise in dP image

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Benchmarking the CH-TLI setup

<table>
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<tr>
<th>System</th>
<th>CH-TLI</th>
<th>Birnbacher et al. [2016]</th>
<th>Michel et al. [2013]</th>
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<tbody>
<tr>
<td>d [cm]</td>
<td>4.35</td>
<td>85.7</td>
<td>15.9</td>
</tr>
<tr>
<td>p₂ [μm]</td>
<td>2</td>
<td>5.4</td>
<td>2.4</td>
</tr>
<tr>
<td>Sensitivity [10⁵]</td>
<td>1.37</td>
<td>9.97</td>
<td>4.16</td>
</tr>
<tr>
<td>Visibility</td>
<td>22%</td>
<td>38.7%</td>
<td>20.7%</td>
</tr>
<tr>
<td>((S_s \cdot ν)_{rel})</td>
<td>1.00</td>
<td>12.8</td>
<td>2.90</td>
</tr>
<tr>
<td>α_{min}</td>
<td>(1.64 \cdot 10^{-7})</td>
<td>(1.7 \cdot 10^{-8}) rad</td>
<td>-</td>
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2. ‘\(d, p₂\)’ the system sensitivity

3. ‘\(ν\)’, the system visibility
### TRANSMISSION VERSUS DIFFERENTIAL PHASE IMAGING

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1. Beta versus delta
2. ‘$d, p_2$’ the system sensitivity
3. ‘$v$’, the system visibility
4. Projection vs differential
2. PROJECTION VERSUS DIFFERENTIAL IMAGING

Transmission

\[ S_{Tr} = \exp(-\mu t) = \exp(-2k\beta t) \]

Differential phase

\[ S_{dP} = \frac{2\pi d}{p_2} \tan \left( \frac{\partial \delta t}{\partial x} \right) \]
2. PROJECTION VERSUS DIFFERENTIAL IMAGING

Transmission

\[ S_{Tr} = \exp(-\mu t) = \exp(-2k\beta t) \]

Differential phase

\[ S_{dp} = \frac{2\pi d}{p_2} \tan \left( \frac{\partial \delta t}{\partial x} \right) \]
Contrast-to-noise metrics are not applicable
So, even theoretically, how to compare $S_{Tr}$ and $dP$?
## Transmission versus Differential Phase Imaging

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1. Beta versus delta
2. ‘\(d, p_2\)’ the system sensitivity
3. ‘\(\nu\)’, the system visibility
4. Projection vs differential
How to quantitatively compare Tr and dP imaging?

Comparing experimental data will be very hard, but even for theoretical data (where the ground truth is known) there is no approach available as we cannot compare $S_{Tr}$ with $S_{dP}$. 
RESEARCH QUESTION

How to quantitatively compare Tr and dP imaging?

Performance metric:
Relative dose required for a lesion to be detectable in Tr and dP

Use virtual studies to benchmark the dP performance against the Tr performance

→ Requires a simulation platform to produce rapidly ‘realistic’ dP and Tr images
— Talbot-Lau interferometry

— A hybrid simulation framework
  - generate ‘realistic’ imagines that match those
    of a TLI scanner

— A detectability study
  - a task-based study
  - human reader studies (4-AFC)

— Application: mammography
Numerical wave propagation

Computationally expensive, not practical for virtual studies where you need a lot of data and large fields of view.

Hybrid image modelling

Combining analytical equations with experimentally measured metrics.
**Expected signal**

\[
S_{Tr} = \exp(-\mu t) = \exp(-2k\beta t)
\]

\[
S_{dp} = \frac{2\pi d}{p_2} \tan\left(\frac{\partial \delta t}{\partial x}\right)
\]

**Expected noise level**

\[
\sigma_{Tr} = \frac{S_{Tr}}{\sqrt{PV}} \sqrt{1 + \frac{1}{S_{Tr}}}
\]

\[
\sigma_{dp} = \frac{1}{\sqrt{PV}} \frac{2}{v^2} \left(1 + \frac{1}{S_{Tr}}\right) \left(1 + \frac{1}{S_{Tr}D^2}\right)
\]

Chabior et al. [2012]
HYBRID IMAGE MODELLING

\[ S_{Tr} \]

Detector and focal spot blur

\[ \mathcal{F}^{-1}\{\mathcal{F}\{S\} \cdot MTF \cdot G_{FS}\} \]

Image

\[ S + N \]

Expected signal

\[ \sigma_{Tr} \]

Correlate and scale noise

\[ \mathcal{F}^{-1}\{\mathcal{F}\{R\} \cdot \sqrt{NPS}\} \cdot \sigma \]
HYBRID IMAGE MODELLING

\[ S_{Tr} = \exp(-\mu t) \]
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**INTRODUCTION**

**TLI**

**SIMULATIONS**

**DETECTABILITY STUDY**

**APPLICATIONS**

**CONCLUSION**
HYBRID IMAGE MODELLING

\[ S_{Tr} \rightarrow \mathcal{F}^{-1}\{\mathcal{F}\{S\} \cdot \text{MTF} \cdot G_{FS}\} \rightarrow \text{Correlate and scale noise} \rightarrow S + N \]

- Expected signal: \( S_{Tr} \)
- Detector and focal spot blur: \( \mathcal{F}^{-1}\{\mathcal{F}\{S\} \cdot \text{MTF} \cdot G_{FS}\} \)
- Expected noise: \( \sigma_{Tr} \)
- Correlate and scale noise: \( \mathcal{F}^{-1}\{\mathcal{F}\{R\} \cdot \sqrt{NPS}\} \cdot \sigma \)

MTF: measured
\( G_{FS} \): analytical
**HYBRID IMAGE MODELLING**

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\[
\begin{align*}
\text{Expected signal} & \quad S_{Tr} \quad \xrightarrow{\mathcal{F}\{-1\} \{\mathcal{F}\{S\} \cdot MTF \cdot G_{FS}\}} \quad \text{Image} \\
\text{Expected noise} & \quad \sigma_{Tr} \quad \xrightarrow{\mathcal{F}\{-1\} \{\mathcal{F}\{R\} \cdot \sqrt{NPS}\} \cdot \sigma} \quad S + N \\
\end{align*}
\]

R = random generated values with a zero mean and a unit variance
**HYBRID IMAGE MODELLING**

\[
\sigma_{Tr} = \frac{S_{Tr}}{\sqrt{PV}} \sqrt{1 + \frac{1}{S_{Tr}}} \quad \text{NPS: measured PV: measured}
\]

- **Expected signal**
  - \(S_{Tr}\)  
  - \(\mathcal{F}^{-1}\{\mathcal{F}\{S\} \cdot MTF \cdot G_{FS}\}\)  
  - Image  

- **Expected noise**
  - \(\sigma_{Tr}\)  
  - \(\mathcal{F}^{-1}\{\mathcal{F}\{R\} \cdot \sqrt{NPS}\} \cdot \sigma\)  
  - \(S + N\)

- **Detector and focal spot blur**  
- **Correlate and scale noise**

---

**INTRODUCTION**  ▪ **SIMULATIONS**  ▪ **DETECTABILITY STUDY**  ▪ **APPLICATIONS**  ▪ **CONCLUSION**
HYBRID IMAGE MODELLING

Expected signal

\[ S_{Tr} \]

Detector and focal spot blur

\[ \mathcal{F}^{-1}\{\mathcal{F}\{S\} \cdot MTF \cdot G_{FS}\} \]

Image

\[ S + N \]

Expected noise

\[ \sigma_{Tr} \]

Correlate and scale noise

\[ \mathcal{F}^{-1}\{\mathcal{F}\{R\} \cdot \sqrt{NPS}\} \cdot \sigma \]
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\[ S_{Tr} \]

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Image

\[ S + N \]

Expected noise

\[ \sigma_{Tr} \]

Correlate and scale noise

\[ \mathcal{F}^{-1}\{\mathcal{F}\{R\} \cdot \sqrt{NPS}\} \cdot \sigma \]

\[ -\log() \]
HYBRID IMAGE MODELLING

\[ S_{dp} = \frac{2\pi d}{p^2} \tan \left( \frac{\delta t}{\delta x} \right) \]
HYBRID IMAGE MODELLING

**Expected signal**

\[ S_{dP} \]

\[ \mathcal{F}^{-1}\{\mathcal{F}\{S\} \cdot MTF \cdot G_{FS}\} \]

**Detector and focal spot blur**

\[ \mathcal{F}^{-1}\{\mathcal{F}\{R\} \cdot \sqrt{NPS}\} \cdot \sigma \]

**Expected noise**

\[ \sigma_{dP} \]

**Correlate and scale noise**

\[ S + N \]

MTF: measured

G<sub>FS</sub>: analytical
HYBRID IMAGE MODELLING

The process involves:
- **Expected signal** $S_{dP}$
- **Detector and focal spot blur**: $\mathcal{F}^{-1}\{\mathcal{F}\{S\} \cdot MTF \cdot G_{FS}\}$
- **Expected noise** $\sigma_{dP}$
- **Correlate and scale noise**: $\mathcal{F}^{-1}\{\mathcal{F}\{R\} \cdot \sqrt{NPS}\} \cdot \sigma$

Resulting in the **Image**: $S + N$

\[ R = \text{random generated values with a zero mean and a unit variance} \]
HYBRID IMAGE MODELLING

Expected signal

\[ S_{dP} \]

Detector and focal spot blur

\[ \mathcal{F}^{-1}\{\mathcal{F}\{S\} \cdot MTF \cdot G_{FS}\} \]

Image

\[ S + N \]

Expected noise

\[ \sigma_{dP} \]

Correlate and scale noise

\[ \mathcal{F}^{-1}\{\mathcal{F}\{R\} \cdot \sqrt{NPS}\} \cdot \sigma \]

\[ \sigma_{dP} = \frac{S_{Tr}}{\sqrt{PV}} \sqrt{\frac{2}{v^2} \left( 1 + \frac{1}{S_{Tr}} \right) \left( 1 + \frac{1}{S_{Tr}D^2} \right)} \]

NPS : measured
PV: measured
\[ v : \text{measured} \]
**HYBRID IMAGE MODELLING**

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**Expected signal**

\[ S_{dp} \]

**Detector and focal spot blur**

\[ \mathcal{F}^{-1}\{\mathcal{F}\{S\} \cdot MTF \cdot G_{FS}\} \]

**Image**

\[ S + N \]

---

**Expected noise**

\[ \sigma_{dp} \]

**Correlate and scale noise**

\[ \mathcal{F}^{-1}\{\mathcal{F}\{R\} \cdot \sqrt{NPS}\} \cdot \sigma \]
PMMA sphere

(a) (b) (c) (d) (e) (f)

Tr signal

x [cm] 10^3

0.4 0.6 0.8

0

x [cm] 10^3

dP signal

Sim Exp

0 0.5

0 -0.5

-0.5 0 0.5
In vivo scan mouse
Model is based on segmented uCT data
How to quantitatively compare Tr and dP imaging?
OUTLINE

— Talbot-Lau interferometry

— A hybrid simulation framework

  — generate ‘realistic’ imagines that match those
    of a TLI scanner

— A detectability study

  — a task-based study

  — human reader studies (4-AFC)

— Application: mammography
Relative dose required for a lesion to be detectable

= measure of relative performance

Via a four alternative forced choice study
Four alternative forced choice (4-AFC)

Zhang et al., SPIE proceedings (2016)
Four alternative forced choice (4-AFC)

Zhang et al., SPIE proceedings (2016)
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Zhang et al., SPIE proceedings (2016)

Psychometric curve fit

\[ \%\text{Correct} = 1 - 0.75 \cdot \exp\left(-\left(\frac{\text{dose}}{a}\right)^b\right) \]
Four alternative forced choice (4-AFC)

Psychometric curve fit – threshold at 62.5%

\[
%\text{Correct} = 1 - 0.75 \cdot \exp\left(-\left(\frac{\text{dose}}{a}\right)^b\right)
\]

Zhang et al., SPIE proceedings (2016)
Four alternative forced choice (4-AFC)

Psychometric curve fit – threshold at 62.5%

\[ \%\text{Correct} = 1 - 0.75 \cdot \exp \left( - \left( \frac{\text{dose}}{a} \right)^b \right) \]

Zhang et al., SPIE proceedings (2016)

If you want to do this for every task it is very time consuming. Make it more general.
Definitions FOM

\[ FOM_{Tr} = \frac{\min(I_{Tr}) - \max(I_{Tr})}{\sigma_{Tr}} \]

\[ FOM_{dP} = \frac{\max(\int |S_{dP}|dx)}{\sigma_{dP}} \]

Should scale with detectability
GENERALIZED TASK BASED DETECTABILITY STUDY

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\[
FOM_{dP} = \frac{\max(\int |S_{dP}| dx)}{\sigma_{dP}}
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Should scale with detectability

Only valid for same task shape!
For a certain task shape

1. **Simulate.** Simulate set of Tr and dP images (bg and obj) with signal and noise combinations ranging between undetectable to detectable

2. **FOM.**

3. **4AFC.**

4. **Thresholds.**

5. **EAK(62.5%).**

6. **RP.**
For a certain task shape

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GENERALIZED TASK BASED DETECTABILITY STUDY

For a certain task shape

Transmission

\[ FOM_{Tr} = \frac{\text{min}(I_{Tr}) - \text{max}(I_{Tr})}{\sigma_{Tr}} \]

Diff. Phase

\[ FOM_{dP} = \frac{\max(\int |S_{dP}| dx)}{\sigma_{dP}} \]

1. **Simulate**. Simulate set of Tr and dP images (bg and obj) with signal and noise combinations ranging between undetectable to detectable

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For a certain task shape:

Transmission

```
\[ FOM_{Tr} = \frac{\min(I_{Tr}) - \max(I_{Tr})}{\sigma_{Tr}} \]
```

Diff. Phase

```
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```
For a certain task shape

1. **Simulate**. Simulate set of \(T_r\) and \(dP\) images (bg and obj) with signal and noise combinations ranging between undetectable to detectable.

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4. **Thresholds**. Calculate the threshold \(FOM_{T_r}\) and \(FOM_{dP}\).

5. **EAK(62.5%)**. Calculate the \(EAK_{T_r}\) and \(EAK_{dP}\) for a given application (combination of bg and obj materials) to reach respectively the \(FOM_{T_r}\) and \(FOM_{dP}\).

6. **RP**.

**GENERALIZED TASK BASED DETECTABILITY STUDY**

\[
\begin{align*}
FOM_{T_r} &= \frac{\min(I_{T_r}) - \max(I_{T_r})}{\sigma_{T_r}} \\
FOM_{dP} &= \frac{\max(\int |S_{dP}| dx)}{\sigma_{dP}}
\end{align*}
\]

E.g. for tumor lesion in adipose tissue which EAK required to reach \(FOM_{T_r} = FOM_{T_r 62.5\%}\).

E.g. for tumor lesion in adipose tissue which EAK required to reach \(FOM_{dP} = FOM_{dP 62.5\%}\).
For a certain task shape

**Transmission**

\[ FOM_{Tr} = \frac{\min(I_{Tr}) - \max(I_{Tr})}{\sigma_{Tr}} \]

E.g. for tumor lesion in adipose tissue which EAK required to reach \( FOM_{Tr} = FOM_{Tr62.5\%} \)

**Diff. Phase**

\[ FOM_{dP} = \frac{\max(\int |S_{dP}|dx)}{\sigma_{dP}} \]

E.g. for tumor lesion in adipose tissue which EAK required to reach \( FOM_{dP} = FOM_{dP62.5\%} \)

\[ RP = \frac{EAK_{Tr}(62.5\%)}{EAK_{dP}(62.5\%)} \]

1. **Simulate**. Simulate set of Tr and dP images (bg and obj) with signal and noise combinations ranging between undetectable to detectable

2. **FOM**. Calculate the FOM of each of the images.

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6. **RP**. The relative performance of an application = \( EAK_{Tr}/EAK_{dP} \)
— Talbot-Lau interferometry

— A hybrid simulation framework
  - generate ‘realistic’ imagines that match those
    of a TLI scanner

— A detectability study
  - a task-based study
  - human reader studies (4-AFC)

— Application: mammography
Application 1. Sphere/lesions of different sizes

Lesion
Shaheen E. et al., Med. Phys. 41(8), 2014
APPLICATIONS

Application 1. Sphere/lesions of different sizes

5.3 mm diam
APPLICATIONS: HOMOGENEOUS BG

Application 1. Sphere/lesions of different sizes

Transmission

<table>
<thead>
<tr>
<th>Simulate</th>
<th>FOM</th>
<th>4AFC</th>
<th>Thresholds</th>
<th>EAK(62.5%)</th>
<th>RP</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Differential phase

<table>
<thead>
<tr>
<th>Simulate</th>
<th>FOM</th>
<th>4AFC</th>
<th>Thresholds</th>
<th>EAK(62.5%)</th>
<th>RP</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
</tr>
</tbody>
</table>

8 different FOM values
15 signal present & 45 signal absent per dose
### APPLICATIONS: HOMOGENEOUS BG

**Application 1. Sphere/lesions of different sizes**

1. Simulate.
2. FOM.
3. 4AFC.
4. Thresholds.
5. EAK(62.5%).
6. RP.

<table>
<thead>
<tr>
<th>Transmission</th>
<th>FOM&lt;sub&gt;Tr&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>= 0.2</td>
<td>= 0.37</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Differential phase</th>
<th>FOM&lt;sub&gt;dp&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>= 1.45</td>
<td>= 3.03</td>
</tr>
</tbody>
</table>

**Transmission Imaging**

**Differential phase Imaging**
APPLICATIONS: HOMOGENEOUS BG

**Application 1. Sphere/lesions of different sizes**

1. Simulate.
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3. 4AFC.
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**Transmission**
- 7 trained readers

**Differential phase**
- 7 trained readers

Graphs showing the relationship between FOM and percentage of correct responses for both transmission and differential phase. The transmission graph has a goodness of fit of $R^2 = 0.96$, with $a = 0.29$ and $b = 1.76$. The differential phase graph has a goodness of fit of $R^2 = 0.93$, with $a = 2.34$ and $b = 4.70$. The images in the left and right columns show the simulated images with varying lesion sizes.
APPLICATIONS: HOMOGENEOUS BG

Application 1. Sphere/lesions of different sizes

1. Simulate.
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Transmission

\[ \text{FOM}_{\text{Tr}}(62.5\%) = 0.34 \]

Differential phase

\[ \text{FOM}_{\text{dP}}(62.5\%) = 2.16 \]
APPLICATIONS: HOMOGENEOUS BG

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Transmission

\[ FOM_{Tr}(62.5\%) = 0.34 \]

\[ FOM_{Tr} = \frac{\min(I_{Tr}) - \max(I_{Tr})}{\sigma_{Tr}} \]

<table>
<thead>
<tr>
<th>Background</th>
<th>lesion</th>
<th>EAK(62.5%) [mGy]</th>
</tr>
</thead>
<tbody>
<tr>
<td>adipose</td>
<td>tumour</td>
<td>0.007(1)</td>
</tr>
<tr>
<td>Glandular</td>
<td>tumour</td>
<td>0.030(4)</td>
</tr>
</tbody>
</table>

Differential phase

\[ FOM_{dP}(62.5\%) = 2.16 \]

\[ FOM_{dP} = \frac{\max(\int |S_{dP}|dx)}{\sigma_{dP}} \]

<table>
<thead>
<tr>
<th>Background</th>
<th>lesion</th>
<th>EAK(62.5%) [mGy]</th>
</tr>
</thead>
<tbody>
<tr>
<td>adipose</td>
<td>tumour</td>
<td>0.71(6)</td>
</tr>
<tr>
<td>Glandular</td>
<td>tumour</td>
<td>6.7(5)</td>
</tr>
</tbody>
</table>

Compositions

Hammerstein G. et al., Rad., 130, 1979
### APPLICATIONS: HOMOGENEOUS BG

#### Application 1. Sphere/lesions of different sizes

1. Simulate.
2. FOM.
3. 4AFC.
4. Thresholds.
5. EAK(62.5%).
6. RP.

##### Transmission

<table>
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##### Differential phase

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<th>EAK(62.5%) [mGy]</th>
</tr>
</thead>
<tbody>
<tr>
<td>adipose</td>
<td>tumour</td>
<td>0.0010(2)</td>
</tr>
<tr>
<td>Glandular</td>
<td>tumour</td>
<td>0.0045(7)</td>
</tr>
<tr>
<td>adipose</td>
<td>tumour</td>
<td>0.71(6)</td>
</tr>
<tr>
<td>Glandular</td>
<td>tumour</td>
<td>6.7(5)</td>
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</tbody>
</table>
APPLICATIONS: HOMOGENEOUS BG

Application 1. Sphere/lesions of different sizes

For our system, we do not expect dP to outperform Tr imaging for these tasks.
 APPLICATIONS: HOMOGENEOUS BG

Application 1. Sphere/lesions of different sizes

For our system, we do not expect dP to outperform Tr imaging for these tasks.
APPLICATIONS

Application 2. Mammo

5.3 mm diam

Mammographic background
1. μCT data of mastectomy
2. Thresholding glandular and adipose tissue
3. Selecting appropriate ROI’s
APPLICATIONS

Application 2. Mammo

<table>
<thead>
<tr>
<th>Uniform</th>
<th>Mammographic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adipose</td>
<td>glandular</td>
</tr>
<tr>
<td><strong>EAK_{Tr}</strong></td>
<td>0.0007(1)</td>
</tr>
<tr>
<td><strong>EAK_{dp}</strong></td>
<td>0.71(6)</td>
</tr>
<tr>
<td><strong>RP</strong></td>
<td>0.0010(2)</td>
</tr>
</tbody>
</table>

Relative Performance vs. Adipose, Glandular, Mammo
APPLICATIONS

Application 1 & 2. Discussion

Diff Phase imaging does not outperform Tr imaging for our system setup.

But our system is not the state of the art system.
Diff Phase imaging does not outperform Tr imaging for our system setup.

\[ R_P \propto \left( \frac{d}{p_2 \cdot n} \right)^2 \]

But our system is not the state of the art system.
APPLICATIONS

Application 1 & 2. Discussion

Diff Phase imaging does not outperform Tr imaging for our system setup.

<table>
<thead>
<tr>
<th>System</th>
<th>CH-TLI</th>
<th>Birnbacher et al. [2016]</th>
<th>Michel et al. [2013]</th>
</tr>
</thead>
<tbody>
<tr>
<td>d [cm]</td>
<td>4.35</td>
<td>85.7</td>
<td>15.9</td>
</tr>
<tr>
<td>p2 [μm]</td>
<td>2</td>
<td>5.4</td>
<td>2.4</td>
</tr>
<tr>
<td>Sensitivity [10⁵]</td>
<td>1.37</td>
<td>9.97</td>
<td>4.16</td>
</tr>
<tr>
<td>Visibility</td>
<td>22%</td>
<td>38.7%</td>
<td>20.7%</td>
</tr>
<tr>
<td>((S_s \cdot v)_{rel})</td>
<td>1.00</td>
<td>12.8</td>
<td>2.90</td>
</tr>
<tr>
<td>(\alpha_{min})</td>
<td>(1.64 \cdot 10^{-7})</td>
<td>(1.7 \cdot 10^{-8}) rad</td>
<td>-</td>
</tr>
</tbody>
</table>

\[ RP \propto \left( \frac{d}{p_2 \cdot v} \right)^2 \]

But our system is not the state of the art system.
With reasonable system optimization dP outperforms Tr for some tasks!

Diff phase is specifically promising to detect small lesions in a complex background.
Application 1 & 2. Discussion

With reasonable system optimization dP outperforms Tr for some tasks! However, this is only an approximation

**APPLICATIONS**

- Magnification, different detector and source properties,...
APPLICATIONS

Application 1 & 2. Discussion

Orientation background affects dP performance

\[ S_{dP} = \frac{2\pi d}{p_2} \tan \left( \frac{\partial \delta t}{\partial x} \right) \]

Horizontal oriented bg

Vertical oriented bg

Horizontal structures are not detected in dP
Application 1 & 2. Discussion

Orientation background affects dP performance

Exploit this feature when developing TLI mammo systems because human breast has inherent orientation?
CH-TLI system not good enough, but other systems in the literature might have sufficient system quality for dP to outperform Tr.

But TLI is a promising tool for the detection of small lesions in a complex background.
Computer simulations can be used to quantitatively estimate the feasibility of applications and/or to estimate the required system quality in TLI