QUANTUM REFERENCE FRAMES
FOR SPACE AND TIME

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Joint work with: A. Belenchia, Č. Brukner, E. Castro Ruiz, P. A. Höhn, A. Vanrietvelde

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F. Giacomini, E. Castro Ruiz, Č. Brukner, arXiv:1811.08228, 2018
E. Castro Ruiz et al., arXiv190(?).XXXXX, 2019

Observers in quantum gravity II
Naples, 1-3 July 2019
Reference frames

Space and time are relational

When we describe a physical property, we take a specific point of view
Reference frames

Space and time are relational

When we describe a physical property, we take a specific point of view

Our “rods” and “clocks” are physical systems
Physical systems are ultimately quantum

$$\frac{1}{\sqrt{2}} (|\gamma_1\rangle + |\gamma_2\rangle)$$
Physical systems are ultimately quantum

\[ \frac{1}{\sqrt{2}} (|\gamma_1\rangle + |\gamma_2\rangle) \]

Can we “attach” a reference frame to an object whose state is in a superposition of classical states (in some basis)?
Physical systems are ultimately quantum

\[ \frac{1}{\sqrt{2}} (|\gamma_1\rangle + |\gamma_2\rangle) \]

Can we “attach” a reference frame to an object whose state is in a superposition of classical states (in some basis)?

Quantum reference frames
Reference frames

Physical systems are ultimately quantum

Can we “attach” a reference frame to an object whose state is in a superposition of classical states (in some basis)?

Quantum reference frames

Disclaimer:
Does not describe spacetime fuzziness, classical reference frames which are in a quantum relationship
Outline

QUANTUM REFERENCE FRAMES FOR SPACE

- Overview of the formalism

- Results
  - Frame dependence of entanglement and superposition
  - Extension of the covariance of quantum mechanics
  - Operational definition of rest frame

F. Giacomini, E. Castro Ruiz, Č. Brukner, arXiv:1811.08228, 2018

QUANTUM REFERENCE FRAMES FOR TIME

- Motivation

- Formalism

- Phenomenological consequences
  - Relativity of interactions
  - Superposition of causal orders

E. Castro Ruiz et al., arXiv190(?).XXXXX, 2019
QUANTUM REFERENCE FRAMES
FOR SPACE
No absolute space

(see also Philipp’s talk)
No absolute space

Relational approach: only relative quantities are considered.

No need of an absolute reference frame.

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No absolute space

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Relational approach: only **relative** quantities are considered.

No need of an absolute reference frame.

(see also Philipp’s talk)
Quantum reference frames

Transformation to relative coordinates

\[ x_A \mapsto -q_C \]
\[ x_B \mapsto q_B - q_C \]

\[ e^{\frac{i}{\hbar} \alpha \hat{p}_B} |x\rangle_B = |x - \alpha\rangle_B \]
Transformation to relative coordinates

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arXiv:1712.07207
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\[ e^{i \alpha \hat{p}_B} |x\rangle_B = |x - \alpha\rangle_B \]

\[ \hat{S}_x = \mathcal{P}_{AC} e^{i \frac{\hbar}{\alpha} \hat{x}_A \hat{p}_B} \]

\[ \mathcal{P}_{AC} \hat{x}_A \mathcal{P}_{AC}^\dagger = -\hat{q}_C \]

parity-swap operator

\[ \rho^{(A)}_{BC} = \hat{S}_x \rho^{(C)}_{AB} \hat{S}_x^\dagger \]

arXiv:1712.07207
Relative states

\[ \hat{S}_x = \mathcal{P}_{AC} e^{\frac{i}{\hbar} \hat{x}_A \hat{p}_B} \]

\[ \rho_{BC}^{(A)} = \hat{S}_x \rho_{AB}^{(C)} \hat{S}_x^\dagger \]

A: new reference frame; B: quantum system; C: old reference frame
Relative states

$$\hat{S}_x = \mathcal{P}_{AC} e^{\frac{i}{\hbar} \hat{x}_A \hat{p}_B}$$

Localised state of A

$$\rho_{BC}^{(A)} = \hat{S}_x \rho_{AB}^{(C)} \hat{S}_x^\dagger$$
Relative states

\[ \hat{S}_x = \mathcal{P}_{AC} e^{\frac{i}{\hbar} x_A \hat{p}_B} \]

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Localised state of A

Product state and spatial superposition

\[ \rho_{BC}^{(A)} = \hat{S}_x \rho_{AB}^{(C)} \hat{S}_x^\dagger \]

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Entangled state

A: new reference frame; B: quantum system; C: old reference frame
Relative states

\[ \hat{S}_x = \mathcal{P}_{AC} e^{\frac{i}{\hbar} \hat{x}_A \hat{p}_B} \]

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Product state and spatial superposition

\[ \rho_{BC}^{(A)} = \hat{S}_x \rho_{AB}^{(C)} \hat{S}_x^\dagger \]

Entangled state

EPR state

\[ \int dx |x\rangle_A |x + X\rangle_B \]

A: new reference frame; B: quantum system; C: old reference frame
Relative states

\[ \hat{S}_x = \mathcal{P}_{AC} e^{\frac{i}{\hbar} \hat{x}_A \hat{p}_B} \]

Localised state of A

\[ \rho^{(A)}_{BC} = \hat{S}_x \rho^{(C)}_{AB} \hat{S}_x^\dagger \]

Product state and spatial superposition

\[ \int dx |x\rangle_A |x + X\rangle_B \]

Entangled state

A: new reference frame; B: quantum system; C: old reference frame
Extended covariance

Schrödinger equation in C’s reference frame

\[ i\hbar \frac{d\rho_{AB}^{(C)}}{dt} = \left[ H_{AB}^{(C)}, \rho_{AB}^{(C)}(t) \right] \]

A: new reference frame
B: quantum system
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To change to the frame of A we apply the transformation \( \hat{S} \)

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To change to the frame of A we apply the transformation $\hat{S}$

\[ i\hbar \frac{d\rho^{(A)}_{BC}}{dt} = \left[ H^{(A)}_{BC}, \rho^{(A)}_{BC}(t) \right] \]

\[ \hat{H}^{(A)}_{BC} = \hat{S} \hat{H}^{(C)}_{AB} \hat{S}^{\dagger} + i\hbar \frac{d\hat{S}}{dt} \hat{S}^{\dagger} \]

\[ \hat{\rho}^{(A)}_{BC} = \hat{S} \hat{\rho}^{(C)}_{AB} \hat{S}^{\dagger} \]

The evolution in the new reference frame is unitary.
Extended covariance

Schrödinger equation in C’s reference frame

\[ i\hbar \frac{d\rho^{(C)}_{AB}}{dt} = \left[ H^{(C)}_{AB}, \rho^{(C)}_{AB}(t) \right] \]

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\[ \hat{\rho}^{(A)}_{BC} = \hat{S} \hat{\rho}^{(C)}_{AB} \hat{S}^\dagger \]

The evolution in the new reference frame is unitary.

We define an extended symmetry transformation as:

\[ \hat{S} \hat{H} \left( \{m_i, \hat{x}_i, \hat{p}_i\}_{i=A,B} \right) \hat{S}^\dagger + i\hbar \frac{d\hat{S}}{dt} \hat{S}^\dagger = \hat{H} \left( \{m_i, \hat{x}_i, \hat{p}_i\}_{i=B,C} \right) \]

Quantum rest frame
Lack of an operational definition of spin (Stern Gerlach experiment) in special relativistic quantum mechanics. (Pauli-Lubanski, Wigner-Pryce, Foldy-Wouthuysen, Chakrabarti, Czachor, Fradkin-Good, Fleming,…)
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Spin is unambiguous in the rest frame

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QRFs allow us to transform to the rest frame of a particle in a superposition of velocities.
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QRFs allow us to transform to the rest frame of a particle in a superposition of velocities.

QRF transformation to the rest frame of a quantum particle

\[ \hat{S}_L = \mathcal{P}^{(v)}_{C\tilde{A}} U_{\tilde{A}} (\Lambda_{\pi C}) \]

superposition of Lorentz boosts

F. Giacomini, E. Castro Ruiz, Č. Brukner,
Quantum rest frame

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Spin is unambiguous in the rest frame

QRFs allow us to transform to the rest frame of a particle in a superposition of velocities.

Operational way of finding a covariant spin operator.

\[ \Xi_i = \hat{S}_L(I_C \otimes \sigma_i)\hat{S}_L^\dagger \]

QRF transformation to the rest frame of a quantum particle

\[ \hat{S}_L = \mathcal{P}^{(\nu)}_{CA} U_{\tilde{A}}(\Lambda_{\pi_C}) \]

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QRFs allow us to transform to the rest frame of a particle in a superposition of velocities.

Operational way of finding a covariant spin operator.

\[ \Xi_i = \hat{S}_L(I_C \otimes \sigma_i)\hat{S}^+_L \]

Opens to practical applications.

QRF transformation to the rest frame of a quantum particle

\[ \hat{S}_L = \mathcal{P}_{\tilde{A}}(\mathcal{U}_{\tilde{A}}(\Lambda_{\pi C})) \]

superposition of Lorentz boosts

QUANTUM REFERENCE FRAMES FOR TIME
A simple clock model

\[ \frac{1}{\sqrt{2}} (|E_0\rangle + |E_1\rangle) \]

\[ H_C = E_0 |E_0\rangle\langle E_0| + E_1 |E_1\rangle\langle E_1| \]

\[ t_\perp = \frac{\pi \hbar}{(E_1 - E_0)} \]
Gravitating clocks lead to a non-classical spacetime

$$H = H_A + H_B - \frac{G}{c^4 x} H_A H_B$$

$$\frac{1}{\sqrt{2}}(|E_0\rangle + |E_1\rangle)$$
Gravitating clocks lead to a non-classical spacetime

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Gravitating clocks lead to a non-classical spacetime

\[ H = H_A + H_B - \frac{G}{c^4 x} H_A H_B \]

\[ \Delta t = \frac{G(E_1 - E_0)}{c^4 x} t \]

\[ t_\perp = \frac{\pi \hbar}{(E_1 - E_0)} \]

\[ \frac{1}{\sqrt{2}} (|E_0\rangle + |E_1\rangle) \]

E. Castro Ruiz, FG, C Brukner, PNAS (2017)
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\[ t_\perp = \frac{\pi \hbar}{(E_1 - E_0)} \]

\[ \Delta t = \frac{G(E_1 - E_0)}{c^4 x} t \]

QM with no time parameter?

Option 1: Far-away observer

\( E_1 \) \( E_0 \)

\( x \)

\( R \gg x \)

QM with no time parameter?

Option 1: Far-away observer

\[ E_1 \quad x \quad E_0 \]

\[ R \gg x \]

Option 2: Reference frames for time evolution (this talk)

Can we “stand” on different clocks and describe quantum dynamics from their point of view?
Timeless quantum mechanics

Timeless quantum mechanics

\[ \hat{C} | \Psi \rangle_{ph} = 0 \]

\[ \hat{C} = \sum_{k=1}^{N} \hat{H}_k + \sum_{j<k} \lambda_{jk} \hat{H}_j \hat{H}_k \]

\[ \lambda_{jk} = -\frac{G}{c^4 x_{jk}} \]
Timeless quantum mechanics

\[ |\Psi\rangle_{ph} \propto \int d\alpha e^{i\frac{\hat{C}}{\hbar}\alpha} |\phi\rangle \]

\[ \hat{C} |\Psi\rangle_{ph} = 0 \]

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\[ \hat{C} = \sum_{k=1}^{N} \hat{H}_k + \sum_{j<k} \lambda_{jk} \hat{H}_j \hat{H}_k \]

\[ | \Psi \rangle_{ph} \propto \int da e^{\frac{i}{\hbar} \hat{C}a} | \phi \rangle \]

\[ \lambda_{jk} = -\frac{G}{c^4 x_{jk}} \]

Perspective of clock \( i \)

\[ i \langle t_i | \Psi \rangle_{ph} = | \psi(t_i) \rangle^{(i)} \]

Timeless quantum mechanics

\[ \hat{C} \langle \Psi \rangle_{ph} = 0 \]

\[ \hat{C} = \sum_{k=1}^{N} \hat{H}_k + \sum_{j<k} \lambda_{jk} \hat{H}_j \hat{H}_k \]

\[ |\Psi\rangle_{ph} \propto \int d\alpha e^{i\frac{\hat{C}}{\hbar} \alpha} |\phi\rangle \]

\[ \lambda_{jk} = -\frac{G}{c^4 x_{jk}} \]

Perspective of clock i

\[ i\langle t_i | \Psi \rangle_{ph} = |\psi(t_i)\rangle^{(i)} \]

\[ i\hbar \left( 1 + \sum_{k \neq i} \lambda_{ik} \hat{H}_k \right) \frac{d|\psi(t_i)\rangle^{(i)}}{dt_i} = \left( \sum_{k \neq i} \hat{H}_k + \sum_{j<k} \lambda_{jk} \hat{H}_j \hat{H}_k \right) |\psi(t_i)\rangle^{(i)} \]
Timeless quantum mechanics

\[ \hat{C} |\Psi\rangle_{ph} = 0 \]

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\[ |\Psi\rangle_{ph} \propto \int d\alpha e^{i\hat{C}\alpha} |\phi\rangle \]

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**Perspective of clock i**

\[ i\langle t_i | \Psi \rangle_{ph} = |\psi(t_i)\rangle^{(i)} \]

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\[ \lambda_{ik} \to 0 \quad \text{Clock hamiltonian from far-away observer} \]

**Timeless quantum mechanics**

\[ \hat{C} | \Psi \rangle_{ph} = 0 \quad \hat{C} = \sum_{k=1}^{N} \hat{H}_k + \sum_{j<k} \lambda_{jk} \hat{H}_j \hat{H}_k \]

\[ |\Psi\rangle_{ph} \propto \int d\alpha e^{i\frac{\hat{C}\alpha}{\hbar}} |\phi\rangle \]

\[ \lambda_{jk} = -\frac{G}{c^4 x_{jk}} \]

**Perspective of clock i**

\[ i\langle t_i | \Psi \rangle_{ph} = |\psi(t_i)\rangle^{(i)} \]

\[ i\hbar \left( 1 + \sum_{k \neq i} \lambda_{ik} \hat{H}_k \right) \frac{d|\psi(t_i)\rangle^{(i)}}{dt_i} = \left( \sum_{k \neq i} \hat{H}_k + \sum_{j<k} \lambda_{jk} \hat{H}_j \hat{H}_k \right) |\psi(t_i)\rangle^{(i)} \]

\[ \lambda_{ik} \rightarrow 0 \quad \text{Clock hamiltonian from far-away observer} \]

\[ \lambda_{ik} \text{ small} \quad i\hbar \frac{d|\psi(t_i)\rangle^{(i)}}{dt_i} = \left( \sum_{k \neq i} \tilde{H}_k + \sum_{j<k} \tilde{\lambda}_{jk} \hat{H}_j \hat{H}_k \right) |\psi(t_i)\rangle^{(i)} \]

\[ \tilde{H}_k = \hat{H}_k (1 - \lambda_{ik} \hat{H}_k) \]

\[ \tilde{\lambda}_{jk} = \lambda_{jk} - \lambda_{ij} - \lambda_{ik} \]

Relativity of interactions

\[ \hat{H}^{(i)} = \sum_{k \neq i} \hat{H}_k + \sum_{j < k} \tilde{\lambda}_{jk} \hat{H}_j \hat{H}_k \]

\[ \hat{H}_k = \hat{H}_k (1 - \lambda_{ik} \hat{H}_k) \]

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Perspective of clock 3

\[ \hat{\lambda}_{5k} = 0 \quad \forall k \neq 3 \]

No interactions between clock 5 and the other clocks
Relativity of interactions

\[ \hat{H}^{(i)} = \sum_{k \neq i} \hat{H}_k + \sum_{j < k} \tilde{\lambda}_{jk} \hat{H}_j \hat{H}_k \]

\[ \hat{H}_k = \hat{H}_k (1 - \lambda_{ik} \hat{H}_k) \]

\[ \tilde{\lambda}_{jk} = \lambda_{jk} - \lambda_{ij} - \lambda_{ik} \]

Perspective of clock 3

\[ \tilde{\lambda}_{5k} = 0 \quad \forall k \neq 3 \quad \text{No interactions between clock 5 and the other clocks} \]

Perspective of clock 1

\[ \tilde{\lambda}_{54} \neq 0 \quad \text{Interactions between clock 5 and clocks 2 and 4} \]

\[ \tilde{\lambda}_{52} \neq 0 \]
Introducing the measurement

F Hellmann, M Mondragon, A Perez, C Rovelli PRD (2007)
Introducing the measurement

\[ [\hat{C}, \hat{O}] = 0 \]

Non-evolving quantities?
Restriction of observables?

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Restriction of observables?

Solution: “Purify” the measurement

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Clocks 1 and 2
System S
Ancilla M

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Introducing the measurement

$$[\hat{C}, \hat{O}] = 0$$

Non-evolving quantities?  
Restriction of observables?

Solution: “Purify” the measurement

Clocks 1 and 2

System S

Ancilla M

Previous Hamiltonian

$$\hat{C} = \hat{H}_1 + \hat{H}_2 + \hat{H}_S + \lambda \hat{H}_1 \hat{H}_2 + (1 + \lambda \hat{H}_1) \sum_i \delta(\hat{T}_2 - t_i) \hat{K}^{MS}_i$$
Introducing the measurement

\[
[\hat{C}, \hat{O}] = 0
\]

Non-evolving quantities?
Restriction of observables?

Solution: “Purify” the measurement

Clocks 1 and 2

System S

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Previous Hamiltonian

\[
\hat{C} = \hat{H}_1 + \hat{H}_2 + \hat{H}_S + \lambda \hat{H}_1 \hat{H}_2 + (1 + \lambda \hat{H}_1) \sum_i \delta(\hat{T}_2 - t_i) \hat{K}_i^{MS}
\]

Time of measurement controlled by clock 2

F Hellmann, M Mondragon, A Perez, C Rovelli PRD (2007)
Introducing the measurement

\[ [\hat{C}, \hat{O}] = 0 \]

Non-evolving quantities?
Restriction of observables?

Solution: “Purify” the measurement

Clocks 1 and 2

System S

Ancilla M

Previous Hamiltonian

\[
\hat{C} = \hat{H}_1 + \hat{H}_2 + \hat{H}_S + \lambda \hat{H}_1 \hat{H}_2 + (1 + \lambda \hat{H}_1) \sum_i \delta(\hat{T}_2 - t_i) \hat{K}_i^{MS}
\]

Time of measurement controlled by clock 2

Observable on S and M

F Hellmann, M Mondragon, A Perez, C Rovelli PRD (2007)
Introducing the measurement

\[ [\hat{C}, \hat{O}] = 0 \]

Non-evolving quantities? Restriction of observables?

Solution: “Purify” the measurement

Clocks 1 and 2

System S

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Previous Hamiltonian

\[ \hat{C} = \hat{H}_1 + \hat{H}_2 + \hat{H}_S + \lambda \hat{H}_1 \hat{H}_2 + (1 + \lambda \hat{H}_1) \sum_i \delta(\hat{T}_2 - t_i) \hat{K}_i^{MS} \]

Time dilation factor due to clock 1

Time of measurement controlled by clock 2

Observable on S and M

F Hellmann, M Mondragon, A Perez, C Rovelli PRD (2007)

The gravitational switch

A

B

M Zych, F Costa, I Pikovski, C Brukner (2017)
The gravitational switch
The gravitational switch

M Zych, F Costa, I Pikovski, C Brukner (2017)
The gravitational switch
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M Zych, F Costa, I Pikovski, C Brukner (2017)
The gravitational switch

\[ \tau_A = 2 \quad \tau_B = 2 \]

M Zych, F Costa, I Pikovski, C Brukner (2017)
The gravitational switch

\[ \tau_A = 2 \quad \tau_B = 2 \]

\[ \hat{U}_A \hat{U}_B |\psi\rangle_S |L\rangle_E \]
The gravitational switch

A

B
The gravitational switch

M Zych, F Costa, I Pikovski, C Brukner (2017)
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The gravitational switch
The gravitational switch

$\tau_A = 2 \quad \tau_B = 2$
The gravitational switch

\[ \hat{U}_B \hat{U}_A |\psi\rangle_S |R\rangle_E \]

A \hspace{2cm} B

\[ \hat{U}_A \]

\[ \hat{U}_B \]

\[ \tau_A = 2 \]

\[ \tau_B = 2 \]
The gravitational switch

\[
\frac{(|L\rangle_E + |R\rangle_E)}{\sqrt{2}} |\psi\rangle_S
\]

M Zych, F Costa, I Pikovski, C Brukner (2017)
The gravitational switch

\[ \frac{(|L\rangle_E + |R\rangle_E)}{\sqrt{2}} |\psi\rangle_S \]

\[ \hat{U}_A \hat{U}_B |\psi\rangle_S |L\rangle_E + \hat{U}_B \hat{U}_A |\psi\rangle_S |R\rangle_E \]

M Zych, F Costa, I Pikovski, C Brukner (2017)
Relative localisation of events
Relative localisation of events
Relative localisation of events

\[ \hat{C} = \sum_{i=A,B,C} \hat{H}_i(1 + \hat{\phi}_i) + \sum_{i=A,B} \delta(\hat{T}_i - t^*) \hat{K}_i^S(1 + \hat{\phi}_i) \]

\[ \hat{\phi}_i = -\frac{GM_E}{c^2 \hat{x}_i} \]
Relative localisation of events

\[ \hat{C} = \sum_{i=A,B,C} \hat{H}_i(1 + \hat{\phi}_i) + \sum_{i=A,B} \delta(\hat{T}_i - \hat{t}^*)\hat{K}_i^S(1 + \hat{\phi}_i) \]

\[ \hat{\phi}_i = -\frac{GM_E}{c^2\hat{x}_i} \]

Far-away observer

Distance between E and the clocks
Relative localisation of events

\[ \hat{C} = \sum_{i=A,B,C} \hat{H}_i (1 + \hat{\phi}_i) + \sum_{i=A,B} \delta(\hat{T}_i - t^*) \hat{K}_i^S (1 + \hat{\phi}_i) \]

\[ \hat{\phi}_i = -\frac{GM_E}{c^2\hat{x}_i} \]

From C’s point of view
Relative localisation of events

$$\hat{C} = \sum_{i=A,B,C} \hat{H}_i (1 + \phi_i) + \sum_{i=A,B} \delta(\hat{T}_i - t^*) \hat{K}^S_i (1 + \phi_i)$$

$$\phi_i = -\frac{G M_E}{c^2 \hat{x}_i}$$

From A’s point of view
Summary

Operational and relational formalism for quantum reference frames for space and time.

For space:
Frame-dependence of entanglement and superposition
Generalisation of covariance
Generalisation of the weak equivalence principle (not covered)
Operational definition of the rest frame of a quantum system (relativistic spin)

For time:
Hamiltonian for interacting clocks (with gravitational time dilation)
Relativity of interactions
Superposition of causal orders
Thank you

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F. Giacomini, E. Castro Ruiz, Č. Brukner, arXiv:1811.08228, 2018
E. Castro Ruiz *et al*., arXiv190(?).XXXXX, 2019