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Nature Photonics **3**, 720-723 (2009)

DOI: 10.1038/nphoton.2009.214

Link to the published version of this paper:
http://dx.doi.org/10.1038/nphoton.2009.214

Optimal quantum cloning of orbital angular momentum photon qubits via Hong-Ou-Mandel coalescence

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The orbital angular momentum (OAM) of light, associated with a helical structure of the wavefunction, has a great potential for quantum photonics, as it allows attaching a higher dimensional quantum space to each photon[1,2]. Hitherto, however, the use of OAM has been hindered by its difficult manipulation. Here, exploiting the recently demonstrated spin-OAM information transfer tools[3,4], we report the first observation of the Hong-Ou-Mandel coalescence[5] of two incoming photons having nonzero OAM into the same outgoing mode of a beam-splitter. The coalescence can be switched on and off by varying the input OAM state of the photons. Such effect has been then exploited to carry out the $1 \rightarrow 2$ universal optimal quantum cloning of OAM-encoded qubits[6–8], using the symmetrization technique already developed for polarization[9,10]. These results are finally shown to be scalable to quantum spaces of arbitrary dimension, even combining different degrees of freedom of the photons.

Since the OAM of photons lies in an infinitely dimensional Hilbert space, it provides a natural choice for implementing single-photon *qudits*, the units of quantum information in a higher dimensional space. This can be an important practical advantage, as it allows increasing the information content per photon, and this, in turn, may cut down substantially the noise and losses arising from the imperfect generation and detection efficiency, by reducing the total number of photons needed in a given process. Qudit-based quantum information protocols may also offer better theoretical performances than their qubit equivalents[11,12], while the combined use of different degrees of freedom of a photon, such as OAM and spin, enables the implementation of entirely new quantum tasks[13–15]. Finally, a OAM state of light can be also regarded as an elementary form of optical image, so that OAM manipulation is related to quantum image processing[16].

All these applications are presently hindered by the technical difficulties associated with OAM manipulation. Despite important successes, particularly in the generation and application of OAM-entangled[17–19] and OAM/polarization hyper-entangled photons[13,14], a classic two-photon quantum interference process such as the Hong-Ou-Mandel (HOM) effect[5] has not been demonstrated yet with photons carrying nonzero OAM. In the case of the polarization degree of freedom, this phenomenon has played a crucial role in many recent developments of quantum information, as well as in fundamental studies of quantum nonlocality. It has been for example exploited for the implementation of the quantum teleportation[20,21], the construction of quantum logic gates for quantum information processing[22], the optimal cloning of a quantum state[9,10], and various other applications[23]. Hitherto, none of these applications has been demonstrated with OAM quantum states.

Quantum cloning – making copies of unknown input quantum states – represents a particularly important and interesting example. The impossibility of making perfect copies, established by the "no-cloning" theorem[24], is a fundamental piece of modern quantum theory and guarantees the security of quantum cryptography[25]. Even though perfect cloning cannot be realized, it is still possible to single out a complete positive map which yields an *optimal quantum cloning*[8] working for any input state, that is, *universal*. By this map, an arbitrary, unknown quantum state can be experimentally copied, but only with a cloning fidelity F – the overlap between the copy and the original quantum state – lower than unity. Implementing quantum cloning is useful whenever there is the need to distribute quantum information among several parties. The concept finds application also in the security assessment of quantum cryptography, the realization of minimal disturbance measurements, in enhancing the transmission fidelity over a lossy quantum channel, and in separating the classical and quantum information[8,26]. Optimal quantum cloning machines, although working probabilistically, have been demonstrated experimentally for polarization-encoded photon qubits by stimulated emission[6,7] and by the symmetrization technique[9,10,27]. In the latter method, the bosonic nature of photons, i.e. the symmetry of their overall wavefunction, is exploited within a two-photon HOM coalescence effect. In this process, two photons impinging simultaneously on a beam splitter (BS) from two different input modes have an enhanced probability of emerging along the same output mode, i.e. of coalescing, as long as they are undistinguishable. If the two photons are made distinguishable by their internal quantum state, for example encoded in the polarization π or in other degrees of freedom, the coalescence effect vanishes. Now, if one of the two photons involved in the process is in a given input state to be cloned and the other in a random one, the HOM effect will enhance the probability that the two photons emerge from the BS with the same quantum state, i.e. with successful cloning, when they emerge together along the same output mode of the BS. For qubit states, the ideal success probability of this scheme is $p = 3/4$ (when exploiting both BS exit ports), while the cloning fidelity for successful events is $F = 5/6$, corresponding to the optimal value[27]. The probabilistic feature of this implementation does not spoil its optimality, since it has been proved that the optimal cloning fidelity is the same for any probabilistic procedure[28].

In this paper, we report the first observation of two-photon HOM coalescence interference of photons carrying nonzero OAM. Furthermore, we exploit this result to demonstrate for the first time the $1 \to 2$ universal optimal quantum cloning (UOQC) of the OAM quantum state of a single photon. More specifically, we show that we can optimally clone any qubit state $|\varphi\rangle_{o_2}$ encoded in the photon OAM bidimensional subspace o_2, spanned by the eigenstates $\{|+2\rangle, |-2\rangle\}$ respectively corresponding to an OAM of $+2$ and -2, in units of $\hbar$. A key technical progress which made these results possible is given by the polarization-OAM bidirectional quantum transfer devices that we have recently demonstrated[4], whose working principle is based on the spin-to-orbital optical angular momentum conversion process taking place in the so-called *q-plates*[3].

As we work in a bidimensional subspace of the orbital angular momentum, it is possible to construct a "Poincaré" (or Bloch) sphere for representing the state of an OAM qubit that is fully analogous to the one usually constructed for a polarization qubit[29]. Being $\{|{+2}\rangle, |{-2}\rangle\}$ the basis in the OAM subspace o_2, which can be considered the OAM equivalent of the circular polarization states, we may introduce the following superposition states $|h\rangle = \frac{1}{\sqrt{2}}(|{+2}\rangle + |{-2}\rangle)$, $|v\rangle = \frac{1}{i\sqrt{2}}(|{+2}\rangle - |{-2}\rangle)$, $|a\rangle = \frac{1}{\sqrt{2}}(|h\rangle + |v\rangle)$, and $|d\rangle = \frac{1}{\sqrt{2}}(|h\rangle - |v\rangle)$, which are the OAM equivalent of horizontal/vertical and antidiagonal/diagonal linear polarizations. The OAM eigenstates $|+2\rangle, |-2\rangle$ have the azimuthal transverse pattern of the Laguerre-Gauss (LG) modes, while states $|h\rangle, |v\rangle$ (as well as $|a\rangle, |d\rangle$) have the azimuthal structure of the Hermite-Gauss (HG) modes.

As a first experimental step, we carried out a HOM coalescence enhancement measurement. To this purpose, we prepared the two input photons in a given input OAM degree of freedom (see Fig. 1 for details). The time delay controller device and the input polarizers guarantee also the perfect temporal and polarization matching of the two photons, so as to make them undistinguishable, except possibly for the OAM state. We expect to observe the constructive interference between the two photons only if the OAM contribution to the bosonic wavefunction is also symmetric in the output, that is if the two outgoing photons have the same OAM. Let us note that the BS is not an OAM preserving optical device, as the reflection on the BS inverts the OAM sign. Therefore, the maximal two-photon coalescence is expected to be observed when the input photons carry exactly opposite OAM. Moreover, coalescence is expected when they have the same HG-like state, e.g. $|h\rangle$ or $|v\rangle$. Fig. 2 shows the results of our experiments. As expected, we find a peak

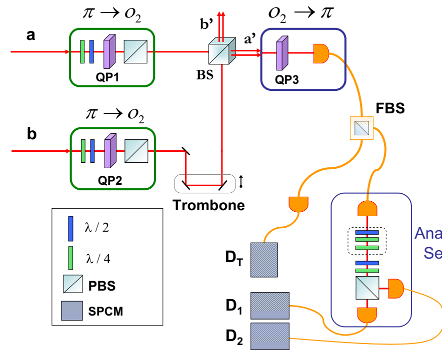

FIG. 1: **Experimental setup for demonstrating the Hong-Ou-Mandel effect and for implementing the OAM quantum cloning.** In the experiments, two photons generated by parametric fluorescence and having the same wavefunction (see Methods for more details) are injected in the two input modes a and b. To set the input OAM state $|\varphi\rangle_{o_2}$ of a photon, the same state is first encoded in the polarization space π, i.e. the state $|\varphi\rangle_\pi$ is prepared by using a combination of a quarter-wave plate ($\lambda/4$) and a half-wave plate ($\lambda/2$), and then the $\pi \to o_2$ quantum transfer device[4] is exploited. In particular, a transferrer composed of a q-plate (QP1 in mode a and QP2 in b) and a polarizer (PBS) filtering the horizontal (H) polarization achieves the transformation $|\varphi\rangle_\pi|0\rangle_o \Rightarrow |H\rangle_\pi|\varphi\rangle_{o_2}$, where $|0\rangle_o$ denotes the zero-OAM state. By this method, the input OAM state of each of the input photons was set independently. The photons in modes a and b then, after a time synchronization controlled by the trombone device, interfere in the balanced beam splitter (BS), giving rise to the HOM effect and/or to the cloning process in the BS output mode a'. In order to analyze the OAM quantum state of the outgoing photons, a $o_2 \to \pi$ transferrer[4] is first used. This device, combining a q-plate and a coupler into a single-mode fiber, achieves the inverse transformation $|H\rangle_\pi|\varphi\rangle_{o_2} \Rightarrow |\varphi\rangle_\pi|0\rangle_o$, thus transferring the quantum information contained in the two photons back into the polarization degree of freedom, where it can be easily read-out. The two photons coupled in the single mode fiber are then separated by a fiber integrated BS (FBS) and detected in coincidence, after analyzing the polarization of one of them by a standard analysis setup. The signal is given by the coincidences between single-photon counting module (SPCM) detectors D_T and either D_1 or D_2, thus corresponding to a post-selection of the sole cases in which both photons emerge from the BS in mode a' and are then split by the FBS.

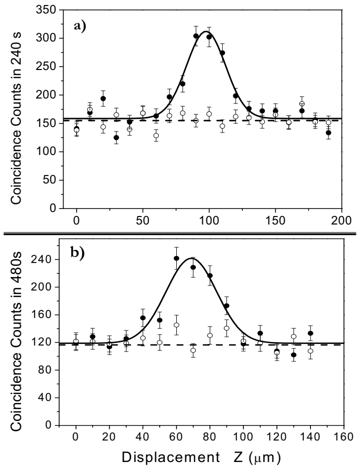

FIG. 2: **Experimental Hong-Ou-Mandel effect with OAM. a**) Input photons having opposite-OAM eigenstates $|+2\rangle$ and $|-2\rangle$ (full circles) and equal-OAM eigenstates $|+2\rangle$ (open circles). The first case leads to coalescence enhancement when the photons are synchronized by the trombone displacement, while the second case shows no enhancement. The solid curve is a best fit based on theoretical prediction. **b**) Input photons having the same HG-like OAM-superposition state $|h\rangle$ (full circles) and orthogonal HG-like states $|h\rangle$ and $|v\rangle$ (open circles). The error bars assume a Poisson distribution and are set at one standard deviation.

in the coincidence counts when the photon arrivals on the beam-splitter are synchronized and the OAM output states are identical. When the OAM output states are orthogonal the coalescence effect is fully cancelled. The mean coincidence-counts enhancement observed in the two Hong-Ou-Mandel experiments is $R = (1.97 \pm 0.05)$, in agreement with the theoretical value of 2.

We now move on to the OAM cloning experiment. Let us first briefly describe the theory of the UOQC process in the OAM subspace o_2. The OAM qubit to be cloned, $|\varphi\rangle_{o_2} = \alpha|+2\rangle + \beta|-2\rangle$, is attributed to the photon in input mode a. The photon in input mode b is prepared in the mixed state described by density matrix $\rho^b_{o_2} = (|+2\rangle\langle+2| + |-2\rangle\langle-2|)/2$. The two photons are then made to interfere in the BS. By selecting the cases in which the two photons emerge from the BS in the same output mode a', the overall two-photon state is then subject to the following projection operator: $P^{a'} = \left(|\Psi^+\rangle^{a'}\langle\Phi^+|^{ab} + |\Phi^+\rangle^{a'}\langle\Psi^+|^{ab} + |\Phi^-\rangle^{a'}\langle\Psi^-|^{ab}\right)$ with $|\Phi^\pm\rangle = 2^{-\frac{1}{2}}(|+2\rangle|+2\rangle \pm |-2\rangle|-2\rangle)$ and $|\Psi^\pm\rangle = 2^{-\frac{1}{2}}(|+2\rangle|-2\rangle \pm |+2\rangle|-2\rangle)$. Note that this projection operator takes into account the fact that the photon OAM undergoes a sign inversion on reflection in the BS. The two photons emerging in mode a' are then separated by means of a second beam splitter and each of them will be cast in the same mixed qubit state

$$\rho^{a'}_{o_2} = \frac{5}{6}|\varphi\rangle_{o_2}\langle\varphi| + \frac{1}{6}|\varphi^\perp\rangle_{o_2}\langle\varphi^\perp| \quad (1)$$

which represents the optimal output of the $1 \rightarrow 2$ cloning process of the state $|\varphi\rangle_{o_2}$, with fidelity $F = {}_{o_2}\langle\varphi|\rho^{a\prime}_{o_2}|\varphi\rangle_{o_2} = \frac{5}{6}$.

Experimentally, the mixed state $\rho^b_{o_2}$ has been prepared by randomly rotating, during each experimental run, a half-wave plate inserted before the q-plate QP2 (see Fig.1). In Table I, we report the experimental

State	Fidelity
$\|h\rangle_{o_2}$	(0.806 ± 0.023)
$\|v\rangle_{o_2}$	(0.835 ± 0.015)
$\|-2\rangle_o$	(0.792 ± 0.024)
$\|+2\rangle_o$	(0.769 ± 0.022)
$\|a\rangle_{o_2}$	(0.773 ± 0.020)
$\|d\rangle_{o_2}$	(0.844 ± 0.019)

TABLE I: **Experimental fidelities for the cloning process.** The experimental values of the fidelity are reported for six specific OAM states.

fidelities of the cloning process for six different input states $|\varphi\rangle_{o_2}$ to verify the universality of the cloning process(see Methods for further details). The measured values are in good agreement with the theoretical prediction $F = \frac{5}{6}$ (see Methods). For the sake of completeness, we have also measured the four Stokes parameters[29] of some cloned states. The results are reported in Fig. 3. Experimentally, the mean length of the vectors on the Bloch's sphere representing the cloned states is found to be $S_{exp} = (0.68 \pm 0.02)$, to be compared with the theoretical value $S_{th} = 2F - 1 = 2/3$. The value of S_{exp} has been estimated as $S_{exp} = \sqrt{S_1^2 + S_2^2 + S_3^2}$, where S_i refers to the i-th measured Stokes component on the Bloch's sphere. Hence, the optimal cloning process corresponds to a shrinking of the whole Bloch sphere, in the subspace o_2: the unitary vector length, related to the visibility of the input qubit, is shortened to two thirds in the output.

Finally, as shown in the Methods section, we note that the symmetrization technique that implements the quantum cloning is optimal not only for qubit states, but also for arbitrary dimension d of the internal spaces of the quantum systems that are cloned (qudits): photons with internal states defined in arbitrarily large subspaces of OAM, or even spanning different internal degrees of freedom at the same time (e.g., polarization, time, arbitrary transverse modes, etc.), can also be cloned optimally by the same method. The fidelity will be given by $F = \frac{1}{2} + \frac{1}{d+1}$,[30] while the success probability decreases only weakly with increasing d, saturating at $p = 1/2$ in the $d \rightarrow \infty$ limit.

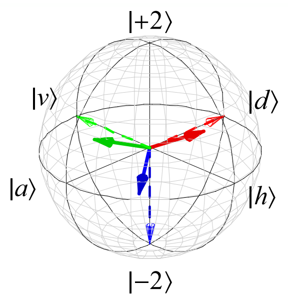

FIG. 3: **Experimental shrunk Bloch sphere of the OAM cloned qubits in the subspace o_2.** The dashed-line arrows refer to the theoretical input qubits, while the solid-line arrows give the experimental output ones.

In conclusion, in this paper we have experimentally carried out the first observation of a two-photon Hong-Ou-Mandel coalescence of photons carrying nonzero OAM and the universal optimal quantum cloning of OAM qubits. These results open the way to the use of OAM in many other quantum information protocols based on two-photon interference effects, such as the generation of complex entangled states (e.g., cluster states), purification processes, and the implementation of logic gates.

Methods

Experimental setup. The input photon pairs are generated via spontaneous parametric fluorescence in a β-barium borate crystal, pumped by the second harmonic of a Ti:Sa mode-locked laser beam. The generated photons have horizontal (H) and vertical (V) linear polarizations, wavelength $\lambda = 795$ nm, and spectral bandwidth $\Delta\lambda = 6$ nm, as determined by an interference filter. The detected coincidence rate of the source is $C_{source} = 5$ kHz. The photons are delivered to the setup via a single-mode fiber, thus defining their transverse spatial mode to a TEM_{00}. The photons are then split by a polarizing beam splitter and injected in the two input modes a and b of the apparatus shown in Fig. 1. The quantum transferrers $\pi \rightarrow o_2$ inserted on the modes a and b are based on q-plates having topological charge $q = 1$, giving rise to the OAM conversion $m \rightarrow m \pm 2$ (in units of $\hbar$) in the light beam crossing it, where the $\pm$ sign is fixed by the input light circular-polarization handedness[3,4]. The conversion efficiency of the q-plates was 0.80 ± 0.05 at 795 nm, due to the reflection on the two faces, not-perfect tuning of the q-plate and birefringence pattern imperfections. The overall transferrer fidelity within the output OAM subspace is estimated at $F_{prep} = (0.96 \pm 0.01)$, mainly due to the imperfect mode quality of the q-plates, leading to a non-perfect $\pi \rightarrow o_2$ conversion. The polarization and temporal matching on the beam splitter between photons on mode a and b has been achieved within an estimated error fixed by the polarization setting accuracy ($0.5°$) and the positioning sensitivity of the trombone device (1μm).

Cloning fidelity and success rate estimation. We set the polarization analysis system so as to have detectors D_1 and D_2 (see Fig. 1) measuring photons respectively in the cloned state $|\varphi\rangle_{o_2}$ and in the orthogonal one $|\varphi^{\perp}\rangle_{o_2}$. The coincidences of either one of these detectors with D_T ensure also the coalescence of the two photons in the same mode. Let C_1, C_2 denote the coincidence counts of D_T and D_1, D_2, respectively. The cloning success rate is then proportional to $C_{tot} = C_1 + C_2$, while the average experimental cloning fidelity is given by $F_{exp} = C_1/C_{tot}$.

The experimental cloning fidelity is to be compared with the prediction that takes into account the imperfect preparation fidelity F_{prep} of the OAM photon state to be cloned (the fidelity of the mixed state is higher than 0.99, due to compensations in the randomization procedure), given by $F_{th} = \frac{F_{prep}R+\frac{1}{2}}{R+1}$, where R is the experimental Hong-Ou-Mandel enhancement. The mean value of the experimental cloning fidelity for all our tests reported in Table I, given by $\bar{F}_{exp} = (0.803 \pm 0.008)$, is indeed consistent with the predicted value $F_{th} = (0.805 \pm 0.007)$.

The experimental coincidence rate C_{tot} can be compared with the predicted one, as determined from C_{source} after taking into account three main loss factors: state preparation probability p_{prep}, successful cloning probability p_{clon}, and detection probability p_{det}. p_{prep} depends on the conversion efficiency of the q-plate and on the probabilistic efficiency of the quantum transferrer $\pi \rightarrow o_2$ (0.5), thus leading to $p_{prep} = 0.40 \pm 0.03$. For ideal input photon states, the experimental success probability of the cloning process on a single BS output mode is expected to be essentially equal to the theoretical one $p_{clon} = 3/8$. The probability p_{det} depends on the q-plate and transferrer efficiencies (0.8×0.5) plus the fiber coupling efficiency $(0.15 - 0.25)$. Hence we have $p_{det} = 0.06 - 0.10$. Therefore, the expected event rate is $C_{source} \times p_{prep}^2 \times p_{clon} \times p_{det}^2 \times \frac{1}{2} = 0.5 - 1.5$ Hz, where the final factor 1/2 takes into account the probability that the two photons are split into different output modes of the analysis FBS. Typically we had around 400 counts in 600s, consistent with the expectations.

In the present experiment, for practical reasons, we adopted a post-selection technique to identify when two photons emerge from the same output mode. In principle, post-selection could be replaced by quantum non-demolition measurements.

Generalization of the cloning process to dimension d. Let us assume that a photon in the unknown input d-dimensional state $|\varphi\rangle$ to be cloned is injected in one arm of a balanced beam splitter (BS), while the "ancilla" photon in the other arm is taken to be in a fully mixed state $\rho = \frac{I_d}{d} = \frac{1}{d}\sum_n |n\rangle\langle n|$, where $|n\rangle$ with $n = 1, ..., d$ is a orthonormal basis. Without loss of generality, we may choose a basis for which $|1\rangle \equiv |\varphi\rangle$. Depending on the state of the ancilla, we must distinguish two cases in the input: (i) the two-photon state is $|1\rangle|1\rangle$, with probability $1/d$, or (ii) it is $|1\rangle|k\rangle$ and $k \neq 1$, with probability $(d-1)/d$. After the interaction in the BS, we consider only the case of two photon emerging in the same BS output mode (case of successful cloning). Then, quantum interference leads to a doubled probability for case (i) than for case (ii), so that the output probabilities are respectively rescaled to $2/(d+1)$ for case (i) and $(d-1)/(d+1)$ for case (ii). The cloning fidelity is 1 for case (i) and $1/2$ for case (ii), so that an overall fidelity of $F = \frac{2}{d+1} \times 1 + \frac{(d-1)}{d+1} \times \frac{1}{2} = \frac{1}{2} + \frac{1}{d+1}$ is obtained, corresponding to the optimal one, as shown in Ref. 30. The success rate of the cloning is $p = \frac{(d+1)}{2d}$.

Author Contribution

E.N., L.S., F.S., F.D.M., L.M., E.S.: Conceived and designed the experiments; E.N., L.S., F.S.: Performed the experiments; E.N., L.S., F.S.: Analyzed the data; L.M., B.P., E.K., E.S.: Contributed materials; E.N., L.S., F.S., F.D.M., L.M., B.P., E.K., E.S.: Paper writing.

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