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Fluctuating-friction molecular motors

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Abstract

We show that the correlated stochastic fluctuation of the friction coefficient can give rise to long-range directional motion of a particle performing a Brownian random walk in the constant landscape of a periodic potential energy. The occurrence of this motion requires the presence of two additional independent bodies interacting with the particle via friction and via the energy potential, respectively, which can move relative to one another. Such a three-body system generalizes the classical Brownian ratchet mechanism, which requires only two interacting bodies. In particular, we describe a simple two-level model of a fluctuating-friction molecular motor that can be solved analytically. In our previous work (Kreuzer M, Marrucci L and Paparo D 2000 *J. Nonlinear Opt. Phys. Mater.* **9** 157) this model was applied, for the first time, in an effort to understand the fundamental mechanism of the photoinduced reorientation of dye-doped liquid crystals. Applications of the same idea to other fields such as molecular biology and nanotechnology can be envisioned. As an example, in this paper we work out a model of the actin–myosin system based on the fluctuating-friction mechanism.

1. Introduction

Driven by the modern tools of molecular biology and by the opening perspectives of nanotechnology, there is currently strong interest in achieving an understanding of all the mechanisms by which systems at a molecular scale can efficiently convert chemical or light energy into mechanical energy. Such systems are often called 'molecular motors', and are at the root of biological processes such as muscle contraction, cell motility, and several intracellular transport processes [1–6]. Unlike ordinary engines, these molecular motors are conceived to work at a single temperature. Moreover they essentially exploit Brownian motion, converting its random behaviour into an ordered directional motion by means of some physical mechanism

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related to energy dissipation. An example of such a mechanism is the so-called 'Brownian ratchet effect' [7–9]. Artificial Brownian ratchets at microscopic scales have been proposed for a variety of technological applications, and in some cases they have been experimentally demonstrated [10–14]. Very recently, the first steps toward engineering truly molecular motors have been taken [15, 16].

Within this field, a specific line of investigation is aimed at developing physical models that capture the essential features of these molecular systems but that are simple enough to be easily understood and studied [9]. Many models are focused on a very simple ideal system: a point-like particle undergoing overdamped Brownian motion under the effect of molecular friction and of a periodic energy potential $U(x)$, where x is the particle coordinate. At a given constant absolute temperature T, friction and Brownian diffusion are characterized by a single quantity that can be taken to be the friction coefficient η or equivalently the diffusion constant $D = kT/\eta$, where k is the Boltzmann constant. In this case, it is well known that even if $U(x)$ is asymmetric for $x \to -x$, as for example in the case of a sawtooth potential, no directional long-range motion can be induced unless the system is driven out of thermodynamic equilibrium by some mechanism [17].

One of the simplest proposed mechanisms for inducing directional motion is a random switching of the system between two internal states characterized by different potentials, say $U_1(x)$ and $U_2(x)$ [18] (see also [19, 20] for a detailed two-state model of kinesin). In other words, the particle is subject to a fluctuating potential, where the fluctuations are described as sudden switches from $U_1(x)$ to $U_2(x)$ and back. These switches are not completely random, but obey stochastic laws in which detailed balance, and hence also thermodynamic equilibrium, is broken. For example, one must assign the probability per unit time $I_{ij}(x)$ of having a nonthermal forced transition from state i to state j, associated with some input of free energy. Moreover, there is the probability per unit time of having spontaneous transitions associated with thermal equilibrium and therefore obeying detailed balance. One may prove for this system that the correlated energy potential fluctuations associated with the random transitions may result in a nonzero average force F acting on the particle and therefore in its drift at a constant average velocity $v = F/\eta$ [18]. This long-range motion is the mechanical output of the motor, whereas the (chemical or optical) free-energy input arises from the nonthermal state transitions. The direction of this motion is dictated by the asymmetry of one or both the potentials $U_i(x)$. However, a directional motion can be induced also by asymmetric transition rates [9].

The possibility of a difference between the friction coefficients (or diffusion constants) experienced by the particle in the two internal states, i.e., $\eta_1 \neq \eta_2$ (or $D_1 \neq D_2$), was considered in previous works but only with a passive role. In particular, if taken alone, this difference cannot give rise to a nonzero average force and velocity of the motor. In other words a fluctuation of friction in the presence of a constant force potential cannot power the motor.

In this paper we show that a stochastic fluctuation of friction induced by asymmetric transition rates can actually give rise to a nonzero average force F. Unexpectedly, however, this steady average force does not generate any long-range motion of the particle, i.e., $v = 0$. More precisely, it can be shown that the average friction force experienced by the particle, owing to the fluctuations in the friction coefficient, does not vanish even for $v = 0$. Therefore, a balance between the average nonzero force arising from the (constant) potential $U(x)$ and the average friction force is established at $v = 0$. At first sight it seems therefore that there is no possibility at all of obtaining motion and mechanical work out of Brownian motion with a fluctuating friction coefficient. We show in the following that this is not completely true.

Let us consider explicitly the two 'external' bodies that interact with the particle and provide respectively the potential forces and the friction forces acting on the particle. In the

following, we call these two bodies the 'sources' of the potential and friction forces. Usually, in discussing molecular motor models, these bodies are considered as large systems that do not move. However, this picture is not always realistic. For example, in biological actin–myosin motors the actin ('thin') filaments (usually considered as the source of potential forces in the framework of these models) are displaced relative to myosin ('thick') filaments by the action of many myosin motors. It is this displacement that is the actual output of the motor. Therefore, we are led naturally to consider also the motion of these 'large' bodies, besides that of the particle itself, in analysing the mechanical output of these systems. The whole motor system must actually be seen then as a three-body dynamical system.

Now, the bodies generating the opposite friction and potential forces acting on the particle are also, by reaction, experiencing opposite nonvanishing average forces. Therefore these two large bodies can be set in motion relative to each other, unless they are blocked by some constraint or they actually coincide. In other words, the motor particle undergoing Brownian motion with a fluctuating friction coefficient can induce a long-range relative displacement of the two bodies with which it is interacting. In the process, the particle itself may also undergo a directional long-range displacement.

This paper is organized as follows. In the next section we describe a simple model of a three-body system that converts chemical (or optical) free energy into mechanical work by exploiting the Brownian motion of a particle that experiences stochastic sudden fluctuations of its friction coefficient between two values. In section 3 we solve analytically the equations of this system for a specific choice of the potential landscape and transition probabilities. We also performed some numerical studies to analyse more general cases. In section 4, in order to show how the idea of a fluctuating-friction molecular motor can be useful in the context of biological systems, we apply our model to the actin–myosin system. Some concluding remarks are given in section 5.

2. The model

Consider three bodies denoted as A, B, and C, corresponding to the motor active unit (the 'particle'), the potential source, and the friction source, respectively (see figure 1). We start working in a frame of reference $\mathcal{R}$ (we may consider it 'fixed', for the sake of simplicity) in which C is motionless while A and B move, their motion being described by coordinates $x_A(t)$ and $x_B(t)$. The motion of B is supposed to be deterministic, i.e., its Brownian fluctuations are neglected, for example because it is a comparatively larger body. For the time being, we assume that B moves at a constant velocity V_B, i.e., $x_B = V_B t$. The motion of the small

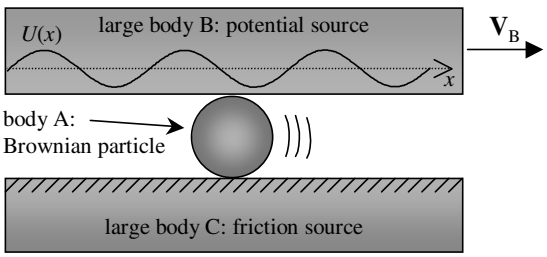

Figure 1. A schematic drawing of our three-body model of a fluctuating-friction molecular motor. The potential $U(x)$ characterizes the interaction between body B and the particle A, where x is their relative position. Body C interacts with the particle via friction.

particle A is instead stochastic, due to Brownian fluctuations. Moreover, we assume that it is overdamped, i.e., inertia of A can be neglected. Therefore, with each internal state $i = 1, 2$ of the motor we may associate an evolving probability density $f_i(x_A, t)$. The interaction between A and B is described by a potential $U_i(x_A, x_B) = U_i(x_A, V_B t)$ that is time dependent, because B is moving. It is therefore convenient to switch to another inertial frame of reference $\mathcal{R}_B$ that is co-moving with B, thus introducing a new relative coordinate $x = x_A - x_B$ in terms of which the potential $U_i(x)$ is stationary.

The stochastic dynamics of body A can then be described by the following pair of coupled Smoluchowski equations:

$$\frac{\partial f_i}{\partial t} + \frac{\partial J_i}{\partial x} = W_i(x) \tag{1}$$

where $J_i(x)$ is the probability current in the frame $\mathcal{R}_B$ and $W_i(x)$ is the net rate of transitions to state i at position x. J_i is given by

$$J_i = -D_i\left(\frac{\partial f_i}{\partial x} + \frac{f_i}{kT}\frac{\partial U_i}{\partial x}\right) - f_i V_B \tag{2}$$

where $D_i = kT/\eta_i$ is the diffusion constant in state i. The first two terms in equation (2) are the standard diffusion and drift currents, respectively. The last term appears because we are working in a frame of reference, $\mathcal{R}_B$, that is moving with respect to the source of friction (normally, overdamped Brownian motion is studied in the 'privileged' reference frame of the fluid providing the friction). Formally, it can be derived starting from equations (1) written in the fixed frame $\mathcal{R}$, by applying the Galilei coordinate transformation $x = x_A - V_B t$ to the $\partial f_i/\partial t$ term. Physically, it describes the drift current induced by the drag force generated by the friction source body C that—in the frame $\mathcal{R}_B$—is moving at constant speed $-V_B$. This term would arise, for example, when describing the Brownian motion of a particle in a viscous fluid that is flowing at constant speed $-V_B$.

For comparing the contributions of fluctuating friction and fluctuating potential, we allow here for both of them to depend on the internal state. For the transition rates we take

$$W_2(x) = -W_1(x) = I(x) f_1(x) - \frac{f_2(x)}{\tau} \tag{3}$$

with τ being the lifetime of state 2 (state 1 is assumed to be stable, while $1/\tau$ is the rate of spontaneous transitions $2 \to 1$). The latter follows from assuming that state transitions are local, i.e., do not involve a variation of x, and thermally induced transitions from level 1 to level 2 are negligible (because the level energy difference is much larger than kT). Note moreover that we are assuming $I(x)$ to depend only on the relative coordinate $x = x_A - x_B$.

Thus far we have made no hypothesis on the periodicity of $U(x)$ and $I(x)$. Actually, this is not a strictly necessary condition in our model, as we will discuss later. However, we assume in the following that the functions $U_i(x)$ and $I(x)$ are periodic, owing to the extended periodic structure of body B. Denoting then by L the half-period in the variable x, we may limit our solution to the interval $x \in [-L, L]$ by imposing the periodic boundary conditions $f_i(-L) = f_i(L)$ and $J_i(-L) = J_i(L)$. The distribution functions f_i are normalized by the condition $\sum_i \int_{-L}^{L} f_i \, dx = 1$.

In the stationary regime, when $\partial f_i/\partial t = 0$, the condition $W_1 = -W_2$ combined with equations (1) implies that the total current $J_t = J_1(x) + J_2(x)$ is independent of x. This first integral can be exploited to reduce equations (1) and (2) to a set of three first-order differential equations in the unknown functions $f_1(x)$, $f_2(x)$, and $J_2(x)$, with J_t playing the role of an eigenvalue. Then, the three remaining periodic boundary conditions together with the normalization condition uniquely determine the unknown functions and the eigenvalue J_t.

In this way, the problem can be easily solved numerically for any functional shape of $U_i(x)$ and $I(x)$ and for a given velocity V_B. For the case of piecewise-linear functions shown in figure 2, the problem can also be solved analytically, as discussed in section 3.

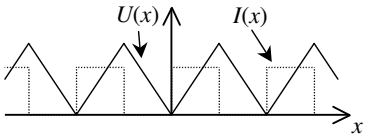

Figure 2. A particular choice of the potential $U(x)$ and transition probability $I(x)$ that allows a fully analytical solution of our model. Notice that, owing to the relative phase of the two functions, the $x \to -x$ symmetry is broken. This symmetry breaking determines the direction of the long-range motion.

Once we have solved the stationary random-walk problem, we are interested in computing the average force F_{AB} that B exerts on A, given by

$$F_{AB} = -\sum_i \int_{-L}^{L} \frac{dU_i}{dx} f_i \, dx \tag{4}$$

and the average friction force F_{AC} arising in the interaction with C, given by

$$F_{AC} = -\sum_i \frac{kT}{D_i} \int_{-L}^{L} (J_i + f_i V_B) \, dx \tag{5}$$

(the latter is easily understood by considering that $J_i + f_i V_B$ is the current in the fixed frame $\mathcal{R}$, i.e., the current relative to body C). By exploiting equations (2), it is straightforward to prove that $F_{AB} + F_{AC} = 0$, reflecting the balance of all forces acting on the element A of the motor. Due to this balance, the particle A does not acquire any momentum (that it could not accumulate, since its inertia is negligible). On the other hand, by interacting with B and C, A mediates a continuous transfer of momentum from the friction source body C to the potential source body B. This transfer is expressed by the reaction force $F_{BA} = -F_{AB} = F_{AC}$ acting on B. In the following, for brevity, we denote the latter just by $F = F_{BA}$. A convenient expression for F, obtained from equation (5), is the following:

$$F(V_B) = -\frac{2kTL}{D_1} \left[J_t + \left(\frac{D_1}{D_2} - 1 \right) \bar{J}_2 \right] - \bar{\eta} V_B \tag{6}$$

where

$$\bar{J}_2 = \frac{1}{2L} \int_{-L}^{L} J_2(x) \, dx \tag{7}$$

is the average current in state 2 and

$$\bar{\eta} = kT \int_{-L}^{L} \left(\frac{f_1(x)}{D_1} + \frac{f_2(x)}{D_2} \right) dx \tag{8}$$

is the average friction coefficient.

The average relative velocity v of the active element A in the frame $\mathcal{R}_B$ can be evaluated as $v = 2LJ_t$. If we set $V_B = 0$ and there are no potential fluctuations, i.e., $U_1(x) = U_2(x) = U(x)$, then $v = J_t = 0$, for any functional form of $U(x)$. To prove this, let us introduce the function

$$g(x) = [D_1 f_1(x) + D_2 f_2(x)] \, e^{U(x)/kT}. \tag{9}$$

The periodic boundary conditions and the periodicity of $U(x)$ imply that $g(-L) = g(L)$. However, from equations (2) we find the relation

$$\frac{\mathrm{d}g}{\mathrm{d}x} = -J_t \mathrm{e}^{U(x)/kT} \tag{10}$$

which after an integration gives

$$J_t = (g(-L) - g(L)) \Big/ \left(\int_{-L}^{L} \mathrm{e}^{U(x)/kT} \, \mathrm{d}x \right) = 0. \tag{11}$$

This means that there can be no long-range displacement of the particle A with respect to the potential source for $V_\mathrm{B} = 0$, as anticipated in the introduction. From equation (6) one can see, however, that the vanishing of J_t does not imply the vanishing of the force F if $D_1 \neq D_2$, as the average current $\bar{J}_2$ does not necessarily vanish for $V_\mathrm{B} = 0$. In fact, as illustrated graphically in figure 3, the system may achieve a stationary nonequilibrium state in which there is a continuous cycling between state 1, in which the particle on average moves one way, and state 2, in which on average the particle moves the opposite way by the same amount. The net particle displacement vanishes but, owing to the different friction coefficients in the two states, the average friction force does not. The actual achievement of this stationary state is proved in section 3 for the particular choice of $U(x)$ and $I(x)$ shown in figure 2. In general, we found numerically that F does not vanish if the nonthermal transition probability $I(x)$ is described by a periodic function that is shifted with respect to the potential $U(x)$ so that the resulting overall transition rates $W_i(x)$ are asymmetric. Note that if one sets $V_\mathrm{B} \neq 0$, then $J_t \neq 0$, i.e., particle A will acquire a directional motion relative to the other two bodies.

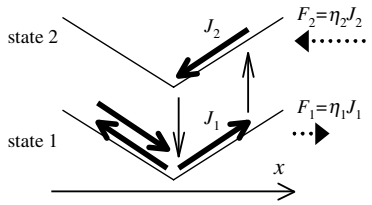

Figure 3. Probability-flux loop due to state transitions and probability currents in the Brownian motion of body A, for the case of no potential fluctuations ($U_1 = U_2 = U(x)$). The potential $U(x)$ and the transition probability $I(x)$ are as in figure 2. Although the total current vanishes, because $J_2 = -J_1$, the contribution to the total friction force $F = F_1 + F_2 = \eta_1 J_1 + \eta_2 J_2$ is nonzero if $\eta_1 \neq \eta_2$.

Let us now come back to body B. Thus far we have simply assumed that B is moving at constant speed V_B, and we have thus determined the average force $F = F(V_\mathrm{B})$ that the motor develops on B itself. Now we can use this information to write a self-consistent equation for the (deterministic) motion of B at steady state:

$$m_\mathrm{B} \frac{\mathrm{d}V_\mathrm{B}}{\mathrm{d}t} = F(V_\mathrm{B}) - \eta_p V_\mathrm{B} - F_\mathrm{ext} = 0 \tag{12}$$

where η_p is an additional friction coefficient associated with the motion of body B, and F_ext is the external load. In particular, setting $V_\mathrm{B} = 0$ in equation (12) we obtain the 'stalling' force $F_\mathrm{ext} = F(0)$. The maximum velocity V_max is instead obtained by solving the force-balance equation with $F_\mathrm{ext} = 0$.

Note that we are assuming here that body B responds only to the average force $F(V_\mathrm{B})$, and not to the instantaneous fluctuating force exchanged with A. As we said, this assumption is justified for example if B is much larger and therefore slower moving than A, so that it

responds only to its time-averaged motion and ignores its fast Brownian fluctuations. Another case that justifies this approach occurs when B interacts simultaneously with many replicas of A and therefore responds only to their total force (proportional to the average force of a single motor). However, it should be noted also that this simplifying assumption of our model is not essential to our main conclusions and that qualitatively similar results are to be expected when Brownian fluctuations of B are taken into account.

One final comment is in order. At first sight, one could be led to believe that by assuming from the start a nonzero velocity for body B we have artificially introduced a bias in the random walk of A, and that it is only this bias that finally leads to the directional motion. That this is not the case is proved by the fact that the internal force $F(V_B)$ does not vanish for $V_B = 0$. It is this force that defines the direction of long-range motion. And it is this internal force that, if not counteracted by a stalling external load, eventually sets body B in motion.

3. Solutions

Let us now describe some specific solutions of our model. It is convenient to use dimensionless quantities, obtained from the corresponding dimensional ones by using $L, L^2/D_2, kT$ as units of length, time, and energy, respectively. We denote a dimensionless quantity by a tilde placed above the corresponding dimensional symbol. In particular $\tilde{I}(x) = I(x)L^2/D_2$, $\tilde{F} = FL/kT$, $\tilde{D}_1 = D_1/D_2$, and $\tilde{V}_B = V_B L/D_2$. For definiteness, let us specialize to the case of triangular-wave potentials and square-wave transition probability shown in figure 2, i.e., $\tilde{U}_i(\tilde{x}) = [2\theta(\tilde{x}) - 1]\tilde{u}_i\tilde{x}$ and $\tilde{I}(\tilde{x}) = \tilde{I}_0\theta(\tilde{x})$, where $\theta(\tilde{x})$ is the unitary step function. In this case all results are analytical. The general expression for $\tilde{F}$ is omitted here for brevity, but we give its linear limit obtained for small excitation probability $\tilde{I}_0$ and velocity $\tilde{V}_B$. The stalling force is

$$\tilde{F}(0) = -\frac{2}{\tilde{D}_1}\tilde{J}_t + 2\left(\frac{1}{\tilde{D}_1} - 1\right)\tilde{\tilde{J}}_2 \tag{13}$$

with

$$\tilde{J}_t = -\frac{\tilde{I}_0\tilde{u}_d\tilde{u}_1^2 e^{-\tilde{u}_1}}{4\Delta^2(1 - e^{-\tilde{u}_1})^2}\left[\Delta + \frac{\cosh(\tilde{u}_1 - \tilde{u}_2/2) - \cosh\delta}{(\sinh\delta)/(2\delta)}\right]$$

$$\tilde{\tilde{J}}_2 = -\frac{\tilde{I}_0\tilde{u}_1\tilde{u}_2\tilde{\tau}}{4\Delta}\left[\frac{1}{\tilde{u}_1\tilde{\tau}} + \frac{\tilde{u}_2}{2} - \frac{\cosh\delta\cosh(\tilde{u}_1/2) - \cosh(\tilde{u}_d/2)}{\sinh(\tilde{u}_1/2)(\sinh\delta)/\delta}\right]$$

where

$$\tilde{u}_d = \tilde{u}_2 - \tilde{u}_1 \qquad \delta = \sqrt{\tilde{u}_2^2/4 + 1/\tilde{\tau}} \qquad \Delta = \tilde{u}_d\tilde{u}_1 + 1/\tilde{\tau}.$$

For $U_1 = U_2$ and $D_1 \neq D_2$ (friction fluctuations only, no potential fluctuations), $\tilde{J}_t = 0$ and the stalling force is proportional to $\tilde{D}_1 - 1 = (D_1 - D_2)/D_2$. Therefore, everything else being fixed, the direction and strength of the force and of the ensuing motion are determined by the variation of the diffusion constant (or friction coefficient) induced by the state fluctuations.

The nonlinear behaviour of the stalling force $\tilde{F}(0)$ versus pumping intensity $\tilde{I}_0$ and lifetime $\tilde{\tau}$ is illustrated in figure 4. The graph shows that there is an optimal $\tilde{\tau}$, i.e., an optimal ratio between the excited-state lifetime and diffusion time. This optimal value decreases for increasing $\tilde{I}_0$. This behaviour is similar to that exhibited by the fluctuating-potential models [18].

For nonzero velocity $\tilde{V}_B$, the (linearized) force is

$$\tilde{F}(\tilde{V}_B) = \tilde{F}(0) - \tilde{\eta}_{\text{eff}}\tilde{V}_B \tag{14}$$

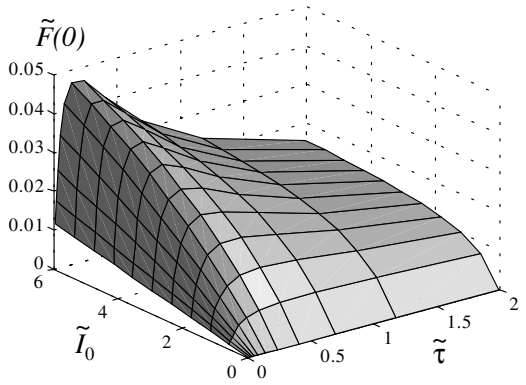

Figure 4. Stalling force $\tilde{F}(0)$ versus pumping intensity $\tilde{I}_0$ and lifetime $\tilde{\tau}$ (dimensionless quantities), for $\tilde{u}_1 = \tilde{u}_2 = 1$ and $\tilde{D}_1 = D_1/D_2 = 2$.

where

$$\tilde{\eta}_{\text{eff}} = \frac{1}{\tilde{D}_1}\left[1 - \frac{\tilde{u}_1^2 \mathrm{e}^{-\tilde{u}_1}}{(1-\mathrm{e}^{-\tilde{u}_1})^2}\right]. \tag{15}$$

Therefore the maximum velocity of the motor is

$$\tilde{V}_{\max} = \frac{\tilde{F}(0)}{(\tilde{\eta}_{\text{eff}} + \tilde{\eta}_p)}. \tag{16}$$

For $U_1 \neq U_2$ and $D_1 = D_2$ (potential fluctuations only, no friction fluctuations), a nonzero current $\tilde{J}_t$ is established, i.e., a long-range displacement of body A with respect to body B occurs. If body B is also fixed (i.e., it is linked to body C), this case reduces to the 'standard' one discussed for example in reference [18]. The average speed of body A will then be $\tilde{v} = 2\tilde{J}_t = -\tilde{D}_1\tilde{F}(0)$.

An interesting difference between the two cases of fluctuating potential and fluctuating friction is given by the behaviour of $\tilde{F}(0)$ for large potential energies, i.e., $\tilde{u}_i = u_i/kT \gg 1$. Indeed, when $D_1 = D_2$ the force shows a thermally activated behaviour, with $\tilde{F} \sim \mathrm{e}^{-\tilde{u}_m}$, where $\tilde{u}_m$ is the minimum of $\tilde{u}_1$ and $\tilde{u}_2$. In contrast, when $D_1 \neq D_2$ and $u_1 = u_2$, a term survives that decays only as $\tilde{F} \sim 1/\tilde{u}_1$. This behaviour is evident in figure 5. It can be explained by considering that for $D_1 = D_2$ the force is proportional to the total average current $\tilde{J}_t$. To contribute to $\tilde{J}_t$ the motor moving element must overcome the potential barrier by acquiring sufficient thermal energy, and the probability for this to happen is proportional to $\mathrm{e}^{-\tilde{u}_i}$. In the case of $D_1 \neq D_2$, the force acquires another term proportional to the average current $\tilde{\tilde{J}}_2$ that need not vanish for $\tilde{J}_t = 0$. It corresponds to a circulation of the particle that moves preferentially forward when in state 1 and backward when in state 2. However, the motor does not have to reach the potential maximum and therefore to be thermally activated. This cycling between the two states and the resulting currents are shown schematically in figure 3.

4. Biological applications: actin–myosin motor

We did not develop this model having in mind a specific biological application. Nonetheless, it is possible that basic features of some biological molecular mechanism are captured by our model. The basic working mechanism of important biological molecular motors is still under debate [21–24], so there is room for proposing new models and offering different perspectives

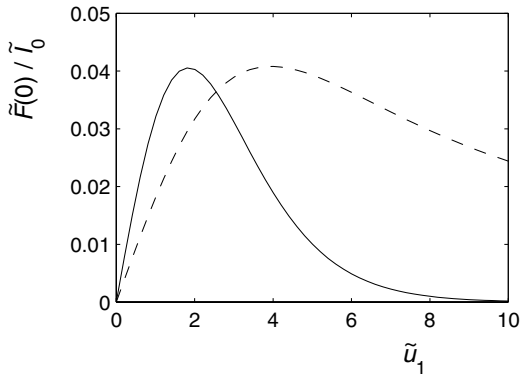

Figure 5. Stalling force $\tilde{F}(0)$ (linearized in $\tilde{I}_0$) versus potential depth $\tilde{u}_1$ for the two cases of fluctuating potential ($\tilde{D}_1 = D_1/D_2 = 1$ and $\tilde{u}_2/\tilde{u}_1 = 2$, solid line) and fluctuating friction ($\tilde{u}_2/\tilde{u}_1 = 1$ and $\tilde{D}_1 = 2$, dashed line). In both examples $\tilde{\tau} = 1$.

on this problem. Therefore we show in the following how our model can be applied to describing a biological molecular motor. However, we stress that we are not attempting here to build a 'realistic' model, our purpose being only to illustrate the basic idea.

To be specific, let us consider the actin–myosin II system—that is, the molecular motor providing the chemomechanical coupling in muscle contraction. When a muscle is contracted, thin filaments of actin and thick filaments of myosin are displaced relative to each other by the concerted action of many myosin 'heads'. A myosin head is a molecular unit linked to the myosin filament through a flexible molecular 'arm' (or 'neck') and interacting via weak intermolecular interactions with the actin filament [25, 26]. A schematic picture of the actin–myosin system is given in figure 6.

In reference [18] the actin–myosin system is modelled by associating the myosin head with the particle (body A), the actin filament with the source of potential (body B), and the surrounding fluid or the actin filament itself with the friction source (body C). The myosin filament is treated as an additional passive 'load' dragged on by the moving myosin head. The potential $U(x)$ describing the adsorption interaction of the head with the actin filament is periodic and asymmetric reflecting the underlying periodic and asymmetric structure of the filament. Attaching and detaching of the head to/from the actin filament are described as sudden changes in the potential landscape.

Here we propose a different set of correspondence rules in order to apply our model to the actin–myosin system, as shown in figure 6. We still identify the Brownian particle (body A) with the myosin head. However, we model here the weak interaction between A and the actin filament as molecular friction, thus identifying the actin filament with body C. In a limit case, the low-friction state allows almost free sliding of A on C, while the high-friction state impedes any sliding. The myosin filament plays instead the role of body B. The potential $U(x)$ is used here to model the deformation energy of the molecular arm connecting the myosin filament with the head. The function $U(x)$ therefore does not need to be periodic in this case, as the limited range of arm deformation constrains the particle A into a single potential well. The periodic potential formalism can be still used, however, by setting infinitely high energy barriers at $\pm L$.

Adenosine triphosphate (ATP) binding to the myosin head and its subsequent hydrolysis triggers the variation of the molecular friction coefficient of the myosin head (from high to low friction). We assume that, owing to some suitable mechanism (e.g. specific intermolecular

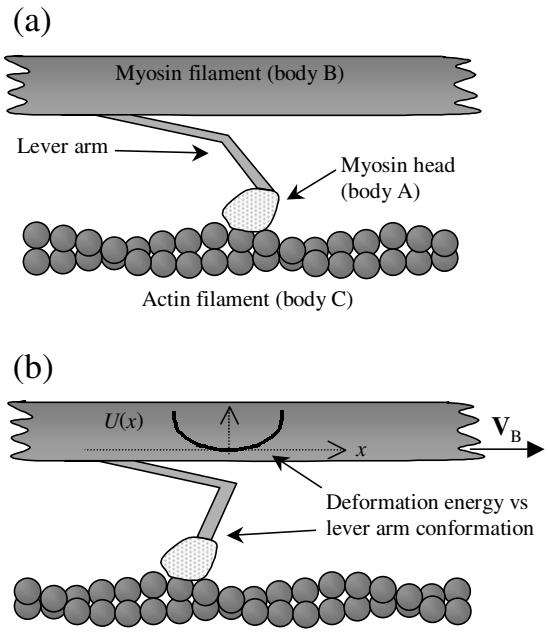

Figure 6. A schematic drawing of the actin–myosin molecular motor. Drawings (a) and (b) refer to two different conformations of the lever arm, corresponding to different values of the coordinate x ($x > 0$ in (a), $x < 0$ in (b)). In (a), also shown are the correspondence rules that link our model (bodies A, B, and C) to the real system. In (b), also shown is the single-well energy $U(x)$ characterizing the deformation of the lever arm versus the deformation coordinate x giving the relative positions of bodies A and B. The 'Brownian power stroke' occurs when the system switches to the high-friction state when $x > 0$ (a) and then relaxes its deformation energy.

interactions), the molecule enzymatic activity is large only when the conformation of the myosin arm is bent in a given direction, for example negative x (figure 6(b)); ATP binding and hydrolysis may then occur only for $x < 0$. This corresponds roughly to having $I(x) > 0$ for $x < 0$ (figure 6(b)) and $I(x) = 0$ for $x > 0$ (figure 6(a)), where the rate I is proportional to the ATP concentration. After an average time τ, the hydrolysis reaction has gone through its full cycle and the motor switches back to a high-friction state.

Given these correspondence rules, our model predicts a continuous directional motion of the myosin motor (head plus filament, i.e., bodies A and B) with respect to the actin filament (body C) (of course this is a relative motion: actually none of the three bodies can be considered fixed). The general behaviour of the motor as a function of external load and ATP concentration is in qualitative agreement with experimental data [27, 28].

We emphasize that no direct causal link between the ATP hydrolysis and conformational transformations is introduced in our model; i.e., the conformational changes, described in our model by the continuous variable x, are not chemically driven but are governed solely by Brownian motion. The asymmetric transition rates allow the elastic energy stored during the Brownian motion to be converted into directed long-range motion. In particular the main 'force-generating step' occurs when the system happens to switch back to the high-friction state for $x > 0$ (figure 6(a)), relaxing then its deformation energy U by moving back to $x \approx 0$. This step may be considered a sort of 'Brownian power stroke'. In contrast, our model assumes no 'chemically driven power stroke'. In this sense, it differs from many other models of biological motors. A power stroke is often modelled either as a sudden change of the elastic constant

characterizing the potential [29], occurring as a consequence of ATP-induced state transitions, or as free sliding on an asymmetric potential as in Brownian ratchet models [18]. On the other hand, the force-generation mechanism of our model in the limit of very high friction in state 2 and very low friction in state 1 becomes similar to that of the old model given by Huxley [1]. A similar sort of 'Brownian power stroke' is assumed also in references [19,20], in connection with the modelling of kinesin.

5. Conclusions

The model presented in this paper was actually developed for a specific application, namely to study the recently discovered strong enhancement of the light-induced molecular reorientation taking place in dye-doped nematic liquid crystals [30]. A more detailed treatment of our modelling of this phenomenon has already been published elsewhere [31], and here we limit ourselves to a brief discussion. The relationship between this optical effect and molecular motors was first pointed out by Palffy-Muhoray and E Weinan, without introducing however the possibility of an active role of fluctuating friction [32]. In our model, dye molecules play the role of motor particles (body A) and the nematic molecular director plays the role of a potential source (body B). The coordinate x is here the angle between the dye molecule orientation and the molecular director, so the motor is a rotary one. The role of the friction source C is played by the translational (centre-of-mass) degrees of freedom of the liquid-crystal molecules themselves. Light powers the motor by continuously promoting internal electronic transitions in the dye molecules, and the final output is a torque (corresponding to the force F) acting on the molecular director and eventually causing its reorientation. There is substantial experimental evidence [33, 34] that the rotational friction coefficients of dye molecules in the two electronic states involved in the transitions are quite different, so the fluctuating-friction mechanism is indeed at work. This proves that this hypothetical mechanism is actually realistic, at least in one specific example.

In this paper, our main goal was to illustrate in a general abstract form, independent of any specific application, the idea that a stochastic fluctuation of a kinetic coefficient, such as friction, mobility, or diffusion constant, can be an effective mechanism for converting chemical (or light) energy into directional motion and mechanical work at the molecular scale, at which Brownian fluctuations dominate. The possibility of extracting work from Brownian motion by means of a suitable modulation of kinetic properties, as opposed to equilibrium potential forces, appears to be new, at least within the field of molecular motors.

We believe that exploitation of this concept could prove profitable in several other fields, besides the nonlinear optics one. In particular, as discussed in section 4, many biological motors are still waiting for a detailed modelling, and any new idea can be very useful in the quest for complete understanding of these complex systems. Similarly, in the field of nanotechnology, the possibility of driving transport via a suitable modulation of the microscopic kinetic coefficients could be an interesting design concept.

In all cases, the relevance of this idea is further enhanced if one takes into account the high structural sensitivity of kinetic coefficients occurring commonly in activated systems, where small changes in the activation energy can lead to huge variations of the kinetic coefficients.

We close by adding that the fluctuating-friction motor idea is not limited to stochastic fluctuations. Just as in the case of fluctuating-potential motors, a periodic modulation in time of the friction coefficient is also expected to induce similar phenomena. The modulation period will then have to be close to the typical diffusion time of the particle, in order to be effective, leading to a new kind of stochastic resonance phenomenon.

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References

[1] Huxley A F 1957 *Prog. Biophys.* **7** 255
[2] Hill T L 1974 *Prog. Biophys. Mol. Biol.* **28** 267
[3] Svoboda K, Schmidt C F, Schnapp B J and Block S M 1993 *Nature* **365** 721
[4] Finer J T, Simmons R M and Spudich R M 1994 *Nature* **368** 113
[5] Yin H *et al* 1995 *Science* **270** 1653
[6] Noji H, Yasuda R, Yoshida M and Kinosita K Jr 1997 *Nature* **386** 299
[7] Magnasco M O 1993 *Phys. Rev. Lett.* **71** 1477
[8] Astumian R D 1997 *Science* **276** 917
[9] Jülicher F, Ajdari A and Prost J 1997 *Rev. Mod. Phys.* **69** 1269
[10] Drexler K E 1992 *Nanosystems: Molecular Machinery, Manufacturing and Computation* (New York: Wiley)
[11] Rousselet J, Salome L, Ajdari A and Prost J 1994 *Nature* **370** 446
[12] Faucheux L P, Bourdieu L S, Kaplan P D and Libchaber A J 1995 *Phys. Rev. Lett.* **74** 1504
[13] Zapata I, Bartussek R, Sols F and Hänggi P 1996 *Phys. Rev. Lett.* **77** 2292
[14] Lee C-S, Jankó B, Derényi I and Barabási A-L 1999 *Nature* **400** 337
[15] Kelly T R, De Silva H and Silva R A 1999 *Nature* **401** 150
[16] Koumura N *et al* 1999 *Nature* **401** 152
[17] Feynman R P, Leighton R B and Sands M 1966 *The Feynman Lectures on Physics* (Reading, MA: Addison-Wesley)
[18] Prost J, Chauwin J-F, Peliti L and Ajdari A 1994 *Phys. Rev. Lett.* **72** 2652
[19] Peskin C S and Oester G 1995 *Biophys. J.* **68** 202
[20] Peskin C S, Ermentrout G B and Oester G 1995 *Cell Mechanics and Cellular Engineering* ed V C Mow *et al* (New York: Springer)
[21] Huxley A F 1998 *Nature* **391** 239
[22] Huxley A F 1998 *Nature* **396** 317
[23] Kitamura K, Tokunaga M, Iwane A H and Yanagida T 1999 *Nature* **397** 129
[24] Cyranoski D 2000 *Nature* **408** 764
[25] Rayment I *et al* 1993 *Science* **261** 50
[26] Rayment I *et al* 1993 *Science* **261** 58
[27] Spudich J A 1994 *Nature* **372** 515
[28] Howard J 1997 *Nature* **389** 561
[29] See for example the discussion in the introduction of
Mogilner A, Mangel M and Baskin R J 1998 *Phys. Lett.* A **237** 297
The model proposed in this paper bears some resemblance to ours, being based on a friction modulation. However, it assumes a driven power stroke as the driving mechanism and neglects the role of Brownian motion.
[30] Jánossy I, Lloyd A D and Wherrett B S 1990 *Mol. Cryst. Liq. Cryst.* **179** 1
[31] Kreuzer M, Marrucci L and Paparo D 2000 *J. Nonlinear Opt. Phys. Mater.* **9** 157
[32] Palffy-Muhoray P and E Weinan 1998 *Mol. Cryst. Liq. Cryst.* **320** 193
[33] Marrucci L, Paparo D, Maddalena P, Massera E, Prudnikova E and Santamato E 1997 *J. Chem. Phys.* **107** 9783
[34] Marrucci L 1998 *Mol. Cryst. Liq. Cryst.* **321** 57