Efficient generation and control of different-order orbital angular momentum states for communication links

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We present an optical scheme to encode and decode 2 bits of information into different orbital angular momentum (OAM) states of a paraxial optical beam. Our device generates the four light angular momentum states of order ± 2 and ± 4 by spin-to-orbital angular momentum conversion in a triangular optical loop arrangement. The switching among the four OAM states is obtained by changing the polarization state of the circulating beam by two quarter-wave plates, and the 2 bit information is transferred to the beam OAM exploiting a single q plate. The polarization of the exit beam is left free for an additional 1 bit of information. The switching among the different OAM states can be as fast as a few nanoseconds, if suitable electro-optical cells are used. This may be particularly useful in communication systems based on light OAM.

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1. INTRODUCTION

Morse code, the first alphabet for electromagnetic communication to be introduced, was based on three symbols: dots, dashes, and spaces. Modern digital communication is instead largely based on binary codes, using only two states of the carrier electromagnetic wave. Different technologies vary, though, for the specific degree of freedom that is modulated for defining such states, such as the wave intensity (or photon number) or its frequency. In the optical domain, the polarization, associated to the vectorial property of the optical field and defining the "spin angular momentum" (SAM) content of a photon, is often considered as a good candidate for free-space telecommunication and quantum communication protocols [1]. The photon SAM is inherently binary, so only 1 bit (in the quantum regime, one qubit) can be encoded in a single photon. Recently, an additional optical degree of freedom associated to the beam phase front, known as the light orbital angular momentum (OAM), received a great deal of attention for various applications in classical and quantum optics [2,3]. Each photon of the beam whose state has an azimuthal phase dependence $\exp(im\varphi)$ (m integer) carries a definite amount of OAM equal to $m\hbar$ per photon. In contrast to SAM, OAM is hence inherently multidimensional, and much more information can be encoded into the OAM of a single photon. Such higher-dimension space can hence be used for expanding the alphabet used in classical and quantum communication [4], as in the Morse code. In the quantum regime, such high-order qubits are generally called "qudits," and their use for quantum information purposes has been shown to have several possible advantages (see, e.g., [5–10]). Photonic qudits have been so far mainly implemented using multiphoton systems or multipath encoding, and the alternative of using OAM encoding has been investigated only very recently. Up to now, single-photon OAM qudits with dimension $d = 3$ ("qutrits") and $d = 4$ ("ququarts") have been generated and employed, e.g., in quantum communication, quantum bit commitment, and quantum key distribution [11–14]. Combined SAM–OAM ququarts have been also recently demonstrated [15]. However, the difficulty and low efficiency of OAM manipulation has so far represented a serious limitation. In particular, current sources of optical OAM are either very rigid (only one OAM value is generated, with no switching or modulation capability) or very inefficient (typically less than 40% of the input photons is converted into the desired OAM modes) and fairly expensive; electro-optical fast manipulation of OAM is virtually nonexistent, while the OAM control flexibility currently provided by spatial light modulators (SLMs) comes at the expense of a slow response ($\sim$1 kHz) and a high cost.

In this paper, we propose a fast, reliable, and inexpensive device to encode classical (or quantum) information into different OAM states of a light beam. The switching among the OAM states can be realized by electro-optical devices, thus ensuring very fast commutation rates. The beam polarization state is not affected and can be further manipulated to store more information. If the SAM is also considered, our device may encode three classical (or quantum qu-)bits of information into a single photon.

2. OPTICAL LOOP DEVICE

The heart of our device is a q plate, a novel optical element made of birefringent liquid crystal spatially oriented in the transverse plane so that it can transfer a well-defined value of topological charge into the output beam depending on the polarization state of the input beam [16,17]. The q plate is characterized by its topological charge q that defines the orientation pattern of the optical axis and its phase retardation δ. When $\delta = \pi$, the q plate is said to be tuned. After tuning, the main effect of the q plate is to convert the SAM of the incident photons into OAM, a process called spin-to-orbital angular momentum conversion [16]. The action of the tuned q plate on the incident photon state is described by [18,19]

$$\widehat{\mathrm{QP}} \cdot |L, m\rangle = |R, m+2\rangle \qquad \widehat{\mathrm{QP}} \cdot |R, m\rangle = |L, m-2\rangle, \tag{1}$$

where $|L\rangle$, $|R\rangle$, and $|m\rangle$ denote the left-circular polarization, the right-circular polarization, and the OAM eigenstate with eigenvalue m, respectively, and $\widehat{\mathrm{QP}}$ is the operator representing the action of the q plate. It is worth noting that we can change the OAM value of the output photons just by switching the input polarization state, which can be accomplished with frequencies restricted only by electro-optical speed limitations ($\sim$ gigahertz). .In our device, the q plate is inserted into a triangular optical loop, as shown in Fig. 1. A polarizing beam splitter (PBS) is used as the input and output gate, so that only the horizontally polarized light can enter and exit from the loop, the vertically polarized light being directly reflected by the PBS. The tuned q plate was sandwiched between two quarter-wave plates (QWPs) and inserted in the loop, as shown in Fig. 1. As will be shown below in Sections 3–5 , respectively, our loop device can do the following:

1. generate the four OAM eigenstates $|\pm 2\rangle$, $|\pm 4\rangle$; the switch among these four states is made acting on the light polarization so that very fast commutation rate can be achieved;
2. generate qubits formed by any pair sorted from the four OAM eigenstates above; the relative amplitudes of the two states forming the qubit is controlled by acting on the light polarization only;
3. generate a state made of the superposition of all OAM eigenstates with even m; the power spectrum of the superposition is controlled by acting on the light polarization only.

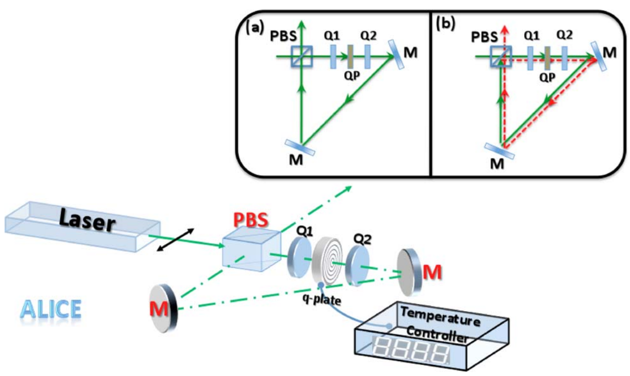

Fig. 1. (Color online) q plate sandwiched between two QWPs (Q_1 and Q_2) inserted in the triangular optical loop. The beam trajectory inside the loop device for the case of $(45°, 45°)$, $(-45°, -45°)$ where the beam passes once [inset (a)], and $(-45°, 45°)$ and $(45°, -45°)$ where the beam passes twice inside the cavity [inset (b)]. The solid green and red lines show the first and second trip, respectively. M, mirror; PBS, polarizing beam splitter; Q, quarter-wave plate; QP, q plate..

3. PURE OAM EIGENSTATES GENERATION

Let us consider a TEM_{00} laser beam with OAM $m = 0$ entering in the optical loop. The output beam is a pure state of order $|m| = 2, 4$ when the QWPs in the loop are set at $\pm 45°$. Let us consider, for example, the case where the two QWPs were set at 45°. The first QWP changes the polarization of the beam circulating in the loop from the horizontal ($|H\rangle$) to the left-circular ($|L\rangle$). The q plate coherently transfers spin-to-OAM, switches the polarization into the right-circular ($|R\rangle$), and provides the beam with an OAM value $m = +2$. The second QWP switches back the right-circular polarization into the horizontal one ($|H\rangle$), so that the light was led out from the loop. Because of the even number of reflections by mirrors, the OAM of the output beam is left to $m = +2$. The full sequence of changes of the photon state is

$$|H,0\rangle \overset{Q_1^{@45^\circ}}{\to} |L,0\rangle \overset{QP}{\to} |R,2\rangle \overset{Q_2^{@45^\circ}}{\to} |H,2\rangle. \tag{2}$$

The same process occurs with the two QWPs set at $-45°$. In this case, however, the output beam is left with $m = -2$. The full sequence is

$$|H,0\rangle \overset{Q_1^{@-45^\circ}}{\to} |R,0\rangle \overset{QP}{\to} |L,-2\rangle \overset{Q_2^{@-45^\circ}}{\to} |H,-2\rangle. \tag{3}$$

Figure 1(a) shows the ray trajectory inside the optical loop for these two cases.

When the two QWPs are set at opposite angles $(45°, -45°)$ or $(-45°, 45°)$, the output beam is left with $m = \pm 4$. Let us consider, for example, the case where the first QWP is set at $+45°$ and the second at $-45°$, respectively. The horizontal polarized beam circulating in the optical loop is changed into the left-circular polarization by the first QWP. The q plate, then, coherently transfers the spin into OAM, and the state changes into $|R, 2\rangle$. The second QWP switches back the polarization into the vertical polarization state, so that the beam is reflected back into the loop by the PBS. However, the sign of the OAM changes due to the odd number of reflections by mirrors and PBS. In the second trip, the first QWP changes the vertical polarization into the right-circular polarization, and the q plate transfers the polarization state into the left-circular polarization and subtracts two to the beam OAM leading to $m = -4$. After that, the second QWP changes back the left-circular polarization into the horizontal polarization so that the beam with $m = -4$ can leave the loop after an even number of reflections by mirrors and PBS. For the $(-45°, 45°)$ configuration, the same process takes place, but the sign of the output OAM is reversed. Inset (b) in Fig. 1 shows the ray trajectory inside the optical loop for the last two cases. The full sequences of changes of the photon states are (M represents here the two mirrors)

$$\begin{aligned} &|H,0\rangle \overset{Q_1^{@45^\circ}}{\to} |L,0\rangle \overset{QP}{\to} |R,2\rangle \overset{Q_2^{@-45^\circ}}{\to} |V,2\rangle \overset{M+\mathrm{PBS}}{\to} |V,-2\rangle \overset{Q_1^{@45^\circ}}{\to} |R,-2\rangle \overset{QP}{\to} \\ &\qquad |L,-4\rangle \overset{Q_2^{@-45^\circ}}{\to} |H,-4\rangle \\ &|H,0\rangle \overset{Q_1^{@-45^\circ}}{\to} |R,0\rangle \overset{QP}{\to} |L,-2\rangle \overset{Q_2^{@45^\circ}}{\to} |V,-2\rangle \overset{M+\mathrm{PBS}}{\to} |V,2\rangle \overset{Q_1^{@-45^\circ}}{\to} |L,2\rangle \overset{QP}{\to} \\ &\qquad |R,4\rangle \overset{Q_2^{@45^\circ}}{\to} |H,4\rangle. \end{aligned} \tag{4}$$

Therefore, the loop device is able to generate -4, -2, $+2$, $+4$ values of OAM by choosing the proper angles for the two

QWPs. Table 1 shows the four possible combinations of QWP angles and the corresponding OAM values of the output beam [20]. In particular, the sign of the output OAM value is fixed by the orientation of the second QWP. It should be noticed, finally, that other OAM eigenstates can be generated by changing the topological charge q of the q plate. More precisely, for a given q, our loop scheme can switch among the four OAM eigenstates $-4q$, $-2q$, $+2q$, $+4q$. One may replace the QWP with electro-optical devices to encode the information in the light beam OAM with switching time of the order of a few nanoseconds. The optical loop setup proposed in this work can be used for classical communications in eight-dimensional SAM–OAM space. As we have already mentioned, an additional classical bit can be encoded in the SAM of the output beam by inserting a further QWP at the exit of the optical loop. So, Alice can transmit to Bob the eight spin-orbit photon states, $(|L\rangle, |R\rangle) \otimes (|-4\rangle, |-2\rangle, |+2\rangle, |+4\rangle)$, corresponding to 3 bits of information per photon. Bob can use, for example, a QWP at 45° followed by a PBS to select the SAM state of the received photons and the holograms shown in Fig. 2 to discriminate the photon OAM [4]. The communication transmitter and receiver scheme shown in Fig. 2 can be fully realized by the available technology. Its main advantage is that 3 bits are encoded in each photon manipulating only the polarization degree of freedom, which can be achieved by very fast and efficient electro-optical switching. The overall efficiency of the system shown in Fig. 2 is very low, however, due to the use of holograms in the detection stage. However, a very efficient transmitter/receiver system can be achieved by replacing the holograms with log-polar phase projectors [21]. The apparatus in Fig. 2 has been intended for classical telecommunication, but it can be applied for single-photon quantum communication too, since the q plate can act as a quantum device [22–24].

4. OAM QUBIT GENERATION

When the orientation angles of the QWPs are set to values different from those reported in Table 1, a superposition of OAM states is generated, in general. In this case, the light is trapped

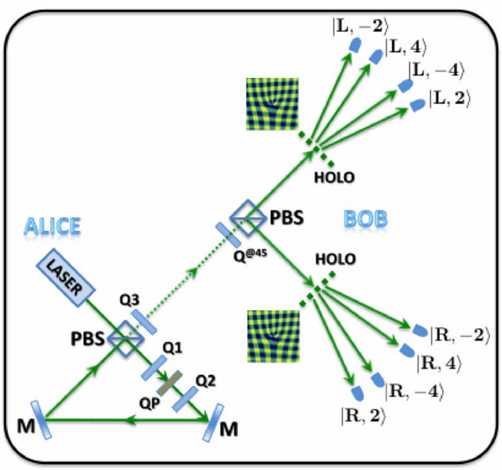

Fig. 2. (Color online) Alice apparatus is similar to what is shown in Fig. 1. In order to encode another information bit, a third QWP (Q_3) is located at the exit face of the optical loop. Alice is able to generate $(|L,-4\rangle, |L,-2\rangle, |L,+2\rangle, |L,+4\rangle, |R,-4\rangle, |R,-2\rangle, |R,+2\rangle \text{and} |R,+4\rangle)$ by setting her QWPs at ±45°. Bob measures the photon SAM state by a suitable QWP and a PBS and measures the photon OAM state by suitable holograms, as shown..

Table 1. Four Possible Combinations of QWP Angles and Their Corresponding Beam's OAM Values

Logical bit	Q_1	Q_2	OAM value
00	+45°	+45°	+2
01	−45°	−45°	−2
10	+45°	−45°	−4
11	−45°	+45°	+4

inside the cavity, making an infinite number of loops. The output state is then given by a superposition of different OAM eigenstates made by the portions of horizontally polarized light exiting the cavity at each loop. In the quantum regime, the superposition is among the probability amplitudes α_N that the photon exits the optical loop after N round trips. When the angle of one of the QWPs inside the cavity is fixed at 45° (or −45°), four different qubits are produced made of any two of the four OAM states $|\pm 2\rangle$, $|\pm 4\rangle$.

More precisely, if the first (second) QWP is fixed at angle 45° the generated output state up to a global phase factor is given by

$$|\psi_1\rangle = C_1(\theta,\psi)[2(\cos(2\theta+\psi) - \sin\psi)|2\rangle - i(1-\sin 2\theta)|\mp 4\rangle], \tag{5}$$

where $C_1(\theta,\psi)$ is a normalization factor depending on the round trip phase delay ψ and on the orientation angle θ of the free QWP. If the first (second) QWP is fixed at −45°, instead, the output state is given by

$$|\psi_2\rangle = C_2(\theta,\psi)[2(\cos(2\theta-\psi) - \sin\psi)|-2\rangle - i(1+\sin 2\theta)|\pm 4\rangle], \tag{6}$$

where $C_2(\theta,\psi)$ is a new normalization factor. It is worth noting that the relative phase of the two OAM eigenstates forming the qubit is fixed to be ±90°, so that only the relative amplitude can be changed by the control parameters θ and ψ. Full control of the relative phase in the qubit could be achieved by inserting in the loop a Dove prism at a variable angle. Equations (5) and (6) have been derived by assuming a laser

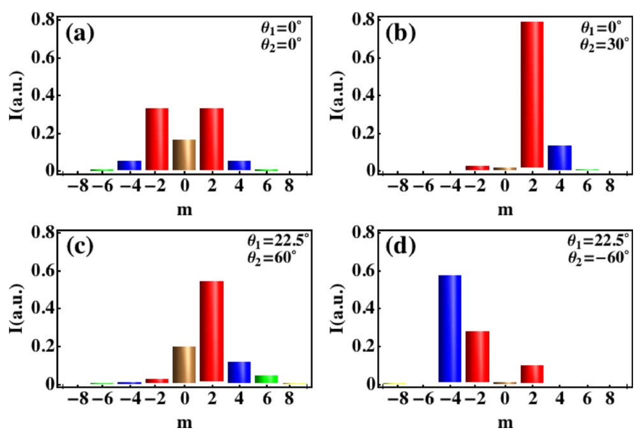

Fig. 3. (Color online) Calculated OAM power spectrum $I_n = |c_{2n}|^2$ of the beam emerging form the loop device for different angles θ_1 and θ_2 of the two QWPs for loop delay $\psi = 0$. The power spectrum can be either (a) symmetric or (b), (c), (d) not symmetric, and (b), (d) the fundamental $m = 0$ component can be suppressed..

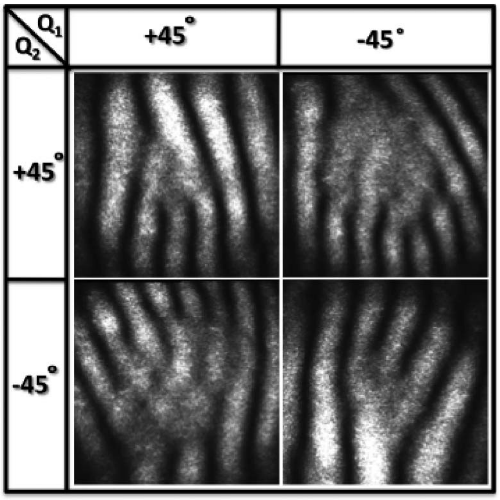

Fig. 4. The interference of the output beam from the loop device and TEM_{00} beam for the four QWP angles shown in Table 1.

coherence length much longer than the loop optical path. The behavior of our loop system with partially coherent light and in the single-photon regime will be the object of future study. However, we think that the possibility of exploiting the photon polarization to control qubits formed by two OAM eigenstates with different m may be useful for quantum computing or other quantum applications.

5. MULTIPLE OAM GENERATION

When both the angles of the QWPs are different from ±45°, a complex superposition of even OAM eigenstates is generated, having the general form $\sum_{n=-\infty}^{+\infty} c_{2n}|2n\rangle$, where c_{2n} depend on the angles θ_1 and θ_2 of the two QWPs and on the loop delay ψ. Figure 3 shows some examples of infinite OAM state superposition obtained for different orientations θ_1 and θ_2 of the two QWPs and for $\psi = 0$. Notice how the symmetry of the OAM power spectrum of the output beam is strongly affected by θ_1 and θ_2. The odd OAM components are missing because we used a $q = 1$ q plate. A full OAM spectrum can be generated by using a $q = 1/2$ q plate, but even in this case the OAM spectrum control is limited, because only a two-parameter subfamily of OAM spectra can be obtained. The possibility of exploiting the light polarization to control full spectra of OAM eigenstates may be useful for future, yet not identified, applications.

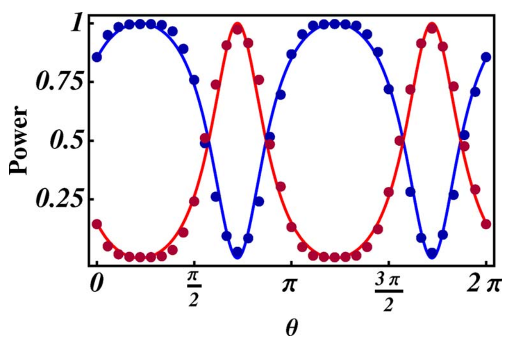

Fig. 5. (Color online) The normalized powers of the $m = 2$ (blue) and $m = -4$ (red) components of the loop output beam as functions of the orientation angle θ of the second QWP. The first QWP was held fixed at 45°. The continuous curve is the fit to the square moduli of the coefficients of modes $m = 2$ and $m = -4$ in Eq. (5). The optical retardation ψ of the loop was used as fitting parameter. In this case , we found a best-fit value $\psi = 0$..

6. EXPERIMENT

In our first experiment, we used a cw TEM_{00} laser source at $\lambda = 532$ nm and measured the output beam phase front by making an interference with a planelike phase front of same frequency. We used an azimuthally oriented liquid crystal homemade q plate. The optical retardation of the q plate was tuned by temperature controller [18] in such a way that it acted as a half-wave plate ($\delta = \pi$). Figure 4 shows the recorded interference pattern of the beam exiting the optical loop for different angles of the QWPs. The absolute value of the OAM is deduced from the number of prongs of the interference fork and the sign from the prongs up or down direction. In our second experiment, we fixed the first QWP at 45° and rotated the second one to generate the qubit formed by the OAM eigenstates 2 and −4 as described before. The alignment of the loop was adjusted by moving the two mirrors to obtain a good symmetric interference pattern [25]. For each angle θ of the second QWP, we measured the power flow associated to the $m = 2$ and $m = -4$ components of the beam exiting the loop device by suitable computer-generated fork holograms displayed onto an SLM. Beyond the hologram, the $m = 0$ component was selected by a pinhole posed in the focal plane of a convergent lens. Finally, the measured power flows were normalized to unity maximum value, and then the normalized power fraction carried by the two modes were compared with the square moduli of the coefficients in Eq. (5). The result is shown in Fig. 5. .The full curve is from Eq. (6). To fit the data, we used the loop retardation ψ as best fit parameter.

7. CONCLUSIONS

We presented a loop device based on a q plate to generate and encode 2 bits of information into the OAM of a single photon. The encoding process is very efficient (nominal efficiency is 100%) and very fast, because it can be fully implemented by electro-optical devices. The encoded information can be read with a computer-generated hologram properly designed to detect all four OAM states simultaneously [4]. The generation process is deterministic, and the setup is suitable for both classical and quantum regimes of light. Furthermore, the optical loop can be easily modified to encode 3 bits of information in a single photon by adding an additional polarization bit. The same setup allows also the generation of qubits made of two different OAM orders or qudits with infinite number of OAM eigenstates. The generation process of single OAM eigenstates, OAM qubits, and OAM qudits with $d = \infty$ is deterministic, has nominal 100% efficiency, and the output OAM state can be switched by very fast electro-optical devices.

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