Polarization effects in the optical reorientation of freely suspended smectic-C liquid-crystal films

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(Received 14 May 1990; accepted for publication 3 October 1990)

A linearly polarized laser beam propagating in a freely suspended smectic-C liquid-crystal film can induce reorientation of the molecular director. A theoretical model is presented which, unlike previous works, takes into account the effects of the polarization changes of the light into the sample. Despite the nonlinearity of the problem, an analytical solution is found. The stability analysis of the trivial solutions has also been carried out. A physical interpretation of the results, some of which are unexpected on the basis of the old models, is given in terms of angular momentum conservation.

I. INTRODUCTION

Nonlinear optical processes in liquid crystals (LC) offer attractive possibilities for low-power optical devices. Their molecules possess a very large dielectric anisotropy and the energy required to change their orientation is very small. This can result in a very large response to optical fields, with associated giant nonlinear effects.[1] For this reason, the possible technical applications of the optical properties of LC are continuously growing.[2] In this context, also the study of the reorientation mechanism underlying the optics of LC has received increasing interest, leading to a number of valuable results, both experimental and theoretical.

Among liquid crystals, the most widely studied for their optical properties are certainly cholesterics and nematics. Not much work has been done regarding the effects of external optical fields on smectics. The optically induced molecular reorientation in the smectic C (SmC) mesophase was observed by Lippel and Young using a linearly polarized laser beam incident onto a freely suspended film of SmC HOAB [4,4bis (heptyloxy)azoxy-benzene]. They reported that large reorientations were observed with an incident optical power of less than 50 mW and presented a simple theory explaining the orientational effect.[3] A more complete theory accounting for the finite width of the laser beam was carried out by Ong, who considered also the presence of a static external magnetic field.[4] In both Ong's and Lippel and Young's models the change of the light polarization in passing through the film was neglected. Ong's calculations confirm the very-low-power threshold required for reorienting SmC freely suspended films. Besides this, freely suspended SmC films are also of fundamental interest, because they can be formed by an integer number of smectic layers, ranging from 2 to 2000, each layer approximately 25 Å thick, so they can show a nearly two-dimensional character. Moreover, they have one reorientational degree of freedom compared to two for nematics, and the suspended film faces present free-boundary conditions. An experiment to examine in detail the optical reorientation of a freely suspended film of SmC LC could be of great interest, because the effect of layer ordering on reorientational nonlinear optics can be investigated and the balance between optical and elastic torques can increase our understanding of the elastic properties of such materials. The experimental results should be eventually compared with a suitable model for the optical reorientation.

In this paper we present a theoretical study of the behavior of a freely suspended SmC film irradiated by a linearly polarized laser beam by taking into account the change of the light polarization in traversing the film. We find that in general the optical torque acting on the LC molecules is different from the one used by Ong in his paper, and this leads to the appearance of new and somewhat unexpected phenomena when the thickness of the film becomes comparable with the optical wavelength. We also provide a physical explanation of these nonintuitive effects based on total (material + radiation) angular momentum conservation.

In the following sections, we first present and discuss the relevant equations governing the reorientation in the smectic film and the change of the light polarization in the plane-wave approximation (Sec. II). In Sec. III, we prove that, although strongly nonlinear, the problem can be worked out analytically and the exact solution is found. In Sec. IV the stability of the undistorted solutions is studied by solving the equations in the small-distortion regime, and the stabilizing effect of the lateral boundary of the film is roughly evaluated.

II. THE TORQUE EQUATION

Smectic C liquid crystals have a layered structure in which the angle θ between the molecular director $\hat{n}$ and the layer normal $\hat{k}$ is fixed. The projection of the director $\hat{n}$ onto the layer plane is called the C-director.[5] Any change in the angle θ by which the director tilts from the layer normal also deforms the layers. Since layer deformation is in part compressional, its characteristic energy is much larger than for an elastic distortion, which leaves the layer thickness intact. A change in the azimuthal angle ϕ made by turning the director around the layer normal $\hat{k}$ leaves the layers intact and other distortions can be neglected. The C-director is a function of the ϕ angle only, viz., $\hat{n}_C = (\cos\phi, \sin\phi, 0)$. When a linearly polarized laser beam is

 0021-8979/91/031269-06$03.00

incident upon such a film along the $\hat{k}$ direction, the induced dipole moment is not parallel to the optical electric field due to the anisotropic nature of the molecules. The optical field then interacts with the induced dipole to produce a torque which can reorient the C-director by varying the azimuthal ϕ angle. This, in turn, produces a change in the direction of the local optical axis in the film, which can affect the polarization of the laser beam traversing the medium, leading to a very complicated nonlinear interaction. In Refs. 1–3, the change of the light polarization due to this feedback mechanism was neglected. As we will show later, this approximation may be satisfactory when the film thickness L is much lower than the optical wavelength λ, but for $L \sim \lambda$ it is certainly inadequate.

In this paper, we consider a linearly polarized laser beam incident onto a freely suspended SmC film along the direction $\hat{k}$ normal to the smectic layers. We assume, moreover, that the laser beam can be treated as a plane wave propagating in a slowly varying uniaxial optical medium, and apply the geometrical optics (slow-envelope) approximation. Light reflection at the film faces and absorption are neglected. Then, it can be shown that the change of the light polarization in traversing the medium is described by the following equations (the z axis is taken along the $\hat{k}$ direction)[6]:

$$\frac{\partial \hat{s}}{\partial z} = \mathbf{\Omega} \times \hat{s}, \tag{1}$$

with $\mathbf{\Omega} = (2\pi \Delta n/\lambda)(\cos 2\phi, \sin 2\phi, 0)$, where ϕ is the azimuthal angle of the director $\hat{n}$, and $\Delta n = n(\theta) - n_o$ is the difference between the refractive indices seen by the extraordinary and the ordinary waves in the medium. The index n_o coincides with the ordinary index of the SmC material, while $n(\theta)$ is given by

$$n(\theta) = \frac{n_o n_e}{(n_e^2 \cos^2\theta + n_o^2 \sin^2\theta)^{1/2}}, \tag{2}$$

n_e being the extraordinary index of the material. For typical smectic C liquid crystals, θ is in the range 10°—30°. In Eq. (1), $\hat{s} = (s_1, s_2, s_3)$ is the unit Stokes vector describing the beam polarization state. The Stokes parameter s_3 is the polarization ellipticity, and s_1 and s_2 are related to the angle ψ formed by the polarization ellipse major axis and the reference x axis by $s_2/s_1 = \tan(2\psi)$. We assume the x axis along the beam polarization direction, so that we have

$$\hat{s}(0) = (1,0,0) \tag{3}$$

at the input plane $z = 0$.

Equation (1) must be solved together with the equation governing the director ϕ angle. This equation has been derived by Zel'dovich *et al.*[7] in the geometric optics approximation and, in the present case, it can be written in the form

$$k_s \frac{\partial^2 \phi}{\partial z^2} + \tau_0 = 0, \tag{4}$$

where τ_0 is the optical torque per unit volume along the $\hat{k}$ direction, given by

$$\tau_0 = (I/c)\Delta n(s_2 \cos 2\phi - s_1 \sin 2\phi). \tag{5}$$

In Eqs. (4) and (5), I is the light intensity (average Poynting vector in the z direction), and k_s is the effective smectic elastic constant, related to the twist-and-bend Franck's elastic constants k_{22} and k_{33} by

$$k_s = (k_{22} \sin^2\theta + k_{33} \cos^2\theta) \sin^2\theta. \tag{6}$$

In the torque equation (4) the dependence of the ϕ angle on the x and y coordinates in the film plane was neglected. This is appropriate if the frame upon which the liquid-crystal film is suspended is much larger than the spot upon which the laser is focused and the intensity is large enough to neglect the anchoring effects at the lateral boundary. We will return, however, to the effects of the lateral boundary in Sec. IV. Finally we note that Eq. (4) applies to steady states only. If time-dependent states are considered, a viscous term $-\gamma(\partial\phi/\partial t)$ (γ = effective viscous coefficient) must be added to the left-hand side of Eq. (4).

Comparison of Eq. (5) with Eq. (1) permits us to rewrite the optical torque in the alternate form:

$$\tau_0 = \frac{I}{\omega}\frac{\partial s_3}{\partial z}, \tag{7}$$

where $\omega = 2\pi c/\lambda$ is the optical frequency. We note that (I/ω) is the photon flux traversing the sample and that $(I/\omega)s_3$ is the angular momentum carried by these photons. Then Eq. (7) shows that the optical torque per unit volume on the LC molecules results from the angular momentum deposited by the photons passing through the planes located at z and $z + dz$. Equation (7) shows also that no torque would result if the incident radiation did not change its original polarization, contrary to what assumed in Refs. 3 and 4. Moreover, when Eq. (7) is inserted into Eq. (4), the latter can be integrated obtaining

$$k_s \frac{\partial \phi}{\partial z} = -(I/\omega)s_3. \tag{8}$$

In deriving this equation we considered the boundary conditions (3) and, in view of their azimuthal anisotropy, we assumed no surface torque at the film faces, viz.,

$$\frac{\partial \phi}{\partial z}(0) = \frac{\partial \phi}{\partial z}(L) = 0. \tag{9}$$

We see that Eq. (8), when evaluated at $z = L$, yields $s_3(L) = 0$, i.e., the light emerging from the sample is still linearly polarized. This can be easily understood in terms of angular momentum conservation, because no net photon angular momentum can be deposited into the sample in steady-state conditions. If this were not the case, in fact, the molecules would be put into collective rotation.[8] As we will see later, however, the polarization of the output beam will be in general rotated of an amount $\Delta\psi$ depending on the laser intensity. Correspondingly, the steady-state distribution of the director across the film will be twisted.

III. THE SOLUTION OF THE TORQUE EQUATION

The torque Eq. (8) and the polarization Eq. (1) form a set of ordinary differential equations that must be solved with the boundary conditions (3) and (9). The solution yields the C-director distribution $\phi(z)$ and the polarization distribution $\hat{s}(z)$ in the film at steady state.

Two trivial solutions satisfying the required boundary conditions are $\phi \equiv 0$ and $\phi \equiv \frac{1}{2}\pi$. These solutions correspond to the propagation of a purely extraordinary and ordinary waves in the film, respectively. The sample remains undistorted and no change of the light polarization occurs.

Besides these solutions, however, others can be found. It is convenient passing to a "rotating" frame, introducing the new unit vector $\hat{p}$ as

$$\hat{p}(z) = \mathsf{R}(z)\,\hat{s}(z),$$

with

$$\mathsf{R}(z) = \begin{pmatrix} \cos 2\phi & \sin 2\phi & 0 \\ -\sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{10}$$

where $\mathsf{R}(z)$ corresponds to a rotation of $2\phi(z)$ along the z axis.[5] A straightforward calculation shows that the components of $\hat{p}$ obey an autonomous set of differential equations:

$$\frac{dp_1}{du} = -\tilde{I}\,\tilde{L}\,p_3 p_2,$$

$$\frac{dp_2}{du} = +\tilde{I}\,\tilde{L}\,p_3 p_1 - p_3, \tag{11}$$

$$\frac{dp_3}{du} = p_2,$$

where $u = z/L$ $(0 \leqslant u \leqslant 1)$ and $\tilde{I}$ and $\tilde{L}$ are given by

$$\tilde{I} = I(\lambda/2\pi)^2/(ck_s\Delta n),$$

$$\tilde{L} = 2\pi\Delta n L/\lambda. \tag{12}$$

We see that the dimensionless quantities $\tilde{I}$ and $\tilde{L}$ are proportional to the light intensity I and the film thickness L, respectively. Equations (11) must be solved with the boundary conditions $\hat{p}(0) = (\cos 2\phi_0, -\sin 2\phi_0, 0)$ and $p_3(1) = 0$, where $\phi_0 = \phi(0)$ is the ϕ angle at the input face of the film.

The solution of Eqs. (11) satisfying the proper boundary conditions at $z = 0$ is

$$p_1 = \cos 2\phi_0 - \tilde{I}\,A^2\,\mathrm{sd}^2(\alpha u|m),$$

$$p_2 = -\sin 2\phi_0\,\mathrm{cd}(\alpha u|m)\,\mathrm{nd}(\alpha u|m), \tag{13}$$

$$p_3 = A\,\mathrm{sd}(\alpha u|m),$$

where $\mathrm{sd}(x|m)$, $\mathrm{cd}(x|m)$, and $\mathrm{nd}(x|m)$ are Jacobi's elliptic functions of argument x and parameter m.[9] In Eqs. (13), A, α, and m are constants given by

$$A = -\frac{\sin 2\phi_0}{\sqrt{\Delta}},$$

$$\alpha = \tilde{L}\sqrt{\Delta}, \tag{14}$$

$$m = \frac{1}{2}\left(1 + \frac{2\tilde{I}\cos 2\phi_0 - 1}{\Delta}\right),$$

with

$$\Delta = (1 + 4\tilde{I}^2 - 4\tilde{I}\cos 2\phi_0)^{1/2}. \tag{15}$$

The value of ϕ_0 is obtained by imposing the last boundary condition $p_3(1) = 0$. This yields the equation

$$\alpha = 2nK(m) \quad (n = 1,2,3,\ldots), \tag{16}$$

where $K(m)$ is the quarter period function. Condition (16) must be considered as a transcendent equation for the unknown angle ϕ_0.

Once ϕ_0 is found from this equation, $p_1(u)$, $p_2(u)$, and $p_3(u)$ are obtained from Eqs. (13) and $s_1(u)$, $s_2(u)$, and $s_3(u)$ are determined by inverting Eqs. (10). Finally, the distribution $\phi(u)$ of the azimuthal angle in the film is given by

$$\phi(u) = \phi_0 - \tilde{L}\,\tilde{I}\,A\int_0^u \mathrm{sd}(\alpha x|m)\,dx$$
$$= \phi_0 - \{\sin^{-1}[\sqrt{m}\,\mathrm{cd}(\alpha u|m)] - \sin^{-1}(\sqrt{m})\}. \tag{17}$$

From these equations, we see that, after having crossed the film, the light emerges still linearly polarized but rotated with respect to the polarization of the incident beam by an angle $\Delta\psi = 2\phi_0 + 2\sin^{-1}(\sqrt{m})$ for n odd and unrotated for n even in Eq. (16). Inside the sample, however, the light polarization is elliptical. The molecular director at the input face of the film is rotated of ϕ_0 with respect to the incoming light polarization and for odd n in Eq. (16) the orientation of the sample is twisted of an angle $\Delta\phi = \phi(1) - \phi_0 = 2\sin^{-1}(\sqrt{m})$. Since ϕ_0 depends on the laser intensity $\tilde{I}$, the rotation angle ϕ_0, the internal twist angle $\Delta\phi$, and the polarization rotation angle $\Delta\psi$ change with the incident intensity.

The solutions of the torque and polarization equations given by Eqs. (13)–(17) correspond to steady states where the sample is reoriented and the light polarization changes across the film. These additional solutions reduce to the trivial undistorted solutions for $m = 0$, $\alpha = \frac{1}{2}n\pi$ $(n = 1,2,3,\ldots)$, and $\phi_0 = 0$ or $\frac{1}{2}\pi$. The states characterized by these values of m, α, A, and ϕ_0 are therefore branching points of the trivial solutions. From Eqs. (14) we see that the branching states occur at critical values of the incident light intensity, given by

$$\tilde{I}_n = \tfrac{1}{2} - (n\pi/\tilde{L})^2 \quad \text{for } \phi_0 = 0$$

and

$$\tilde{I}_n = (n\pi/\tilde{L})^2 - \tfrac{1}{2} \quad \text{for } \phi_0 = \tfrac{1}{2}\pi. \tag{18}$$

We see that bifurcations of the $\phi = 0$ state (extraordinary wave) may occur only if the film optical phase length is $\tilde{L} > \pi$ and at critical intensities $\tilde{I}_n < \frac{1}{2}$. Bifurcations of

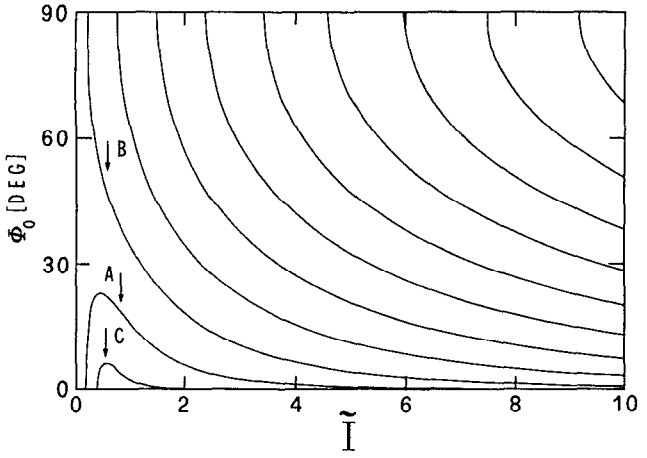

FIG. 1. Rotation angle ϕ_0 vs the normalized light intensity $\tilde{I}$ for an optical sample thickness $\tilde{L} = 2.5\pi$. A, B, and C are the curves originating from the lowest thresholds, respectively.

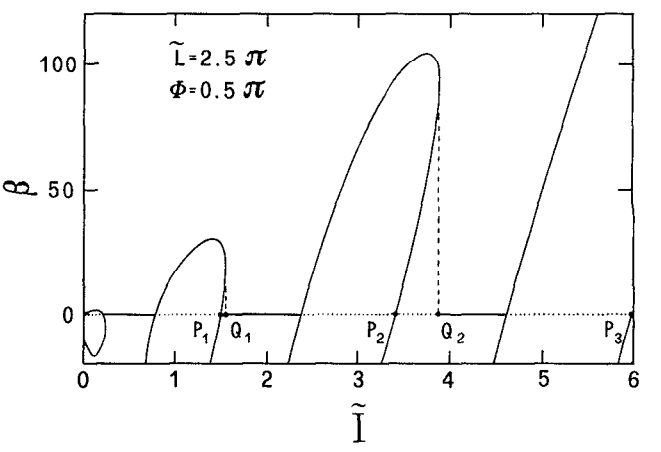

FIG. 3. Plot of the eigenvalues β_n vs the normalized light intensity $\tilde{I}$ for the reference steady state $\phi = \frac{1}{2}\pi$ and optical sample length $\tilde{L} = 2.5\pi$. The instability regions of the reference state are dotted. The P_i's are branching points not implying the stability change of the solution, which occurs indeed at points Q_i.

the $\phi = \frac{1}{2}\pi$ state (ordinary wave) may occur instead for any thickness $\tilde{L}$ and the critical intensities may assume any value.

In Figs. 1 and 2 the rotation angle ϕ_0 and the internal twist angle $\Delta\phi$ are plotted as functions of the reduced intensity $\tilde{I}$ for a sample having optical thickness $\tilde{L} = 2.5\pi$. We notice that, in Fig. 1, the slope of the curves at the branching points at $\phi = 0$ are positive, so that second-order transitions are expected.

IV. STABILITY ANALYSIS

In this section we study the stability of the undistorted states $\phi = 0$ and $\phi = \frac{1}{2}\pi$. As usual, this is made by solving the *time*-dependent torque and polarization equations in the small distortion approximation and by assuming a time dependence of the form $\exp(\beta t)$. Linearization of Eqs. (1) and (4) [with the viscous term $-\gamma\,\partial\phi/\partial t$ included] and the boundary conditions (3) and (9) leads to the following eigenvalue problem for the Liapounov exponent β:

$$(\tilde{L}^2 - x_1^2)x_1 \sin x_1 = (\tilde{L}^2 - x_2^2)x_2 \sin x_2, \tag{19}$$

where x_1 and x_2 are the two roots of the biquadratic algebraic equation

$$x^4 + (\beta - \tilde{L}^2 \pm 2\tilde{I}\,\tilde{L}^2)x^2 - \beta\tilde{L}^2 = 0 \tag{20}$$

lying in the first quadrant of the complex plane. The upper sign in Eq. (20) corresponds to the reference solution $\phi = 0$, the lower sign to $\phi = \frac{1}{2}\pi$. For a fixed $\tilde{I}$, Eq. (19) has a set of real eigenvalues β_n ($n = 1,2,3, \ldots$) with $\beta_{n+1} < \beta_n$. According to Liapounov's method, the reference solution is asymptotically stable for the given $\tilde{I}$ if and only if all β's turn out to be negative. Marginal stability is reached at the critical intensities $\tilde{I}_n$ for which $\beta_n(\tilde{I}_n) = 0$. A simple calculation based on Eqs. (19) and (20) shows that the critical intensities result the same as the ones given by Eqs. (18). Nevertheless, the stability analysis shows some new and somewhat unexpected features.

First, the critical branching points may not coincide with the points where the reference state changes its stability. This is shown in Fig. 3, where the eigenvalues β_n are shown as a function of the reduced intensity $\tilde{I}$ for the reference state $\phi = \frac{1}{2}\pi$. The points P_i in the figure, for example, are branching points where the distorted and undistorted solutions coalesce, but they are also points where the system remains unstable. Stability is reached at higher intensities (points Q_i). The failure of the coincidence between stability switching and branching points was found also in the case of laser-induced reorientation in planarly aligned nematics with strong anchoring at the walls.[10]

Second, the state $\phi = 0$, where the molecular director is uniformly directed along the laser polarization direction, may be unstable for very weak intensities ($\tilde{I} \to 0$). For $\tilde{I} \to 0$, in fact, the eigenvalues β_n of Eqs. (19) and (20) are given by

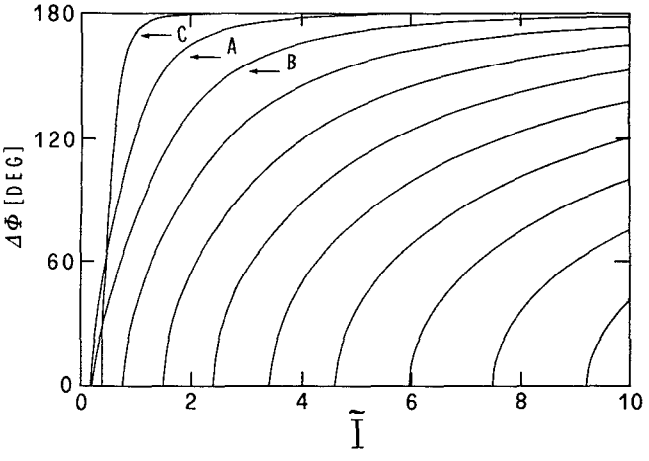

FIG. 2. Twist angle $\Delta\phi$ vs the normalized light intensity $\tilde{I}$ for an optical sample thickness $\tilde{L} = 2.5\pi$. Curves A, B, and C correspond to their analogous in Fig. 1.

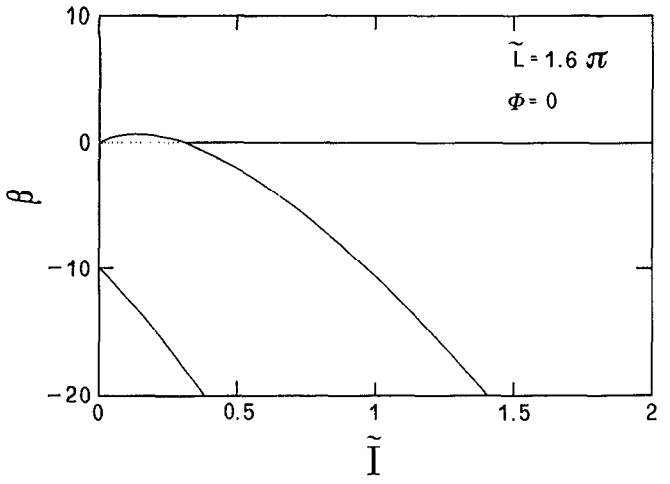

FIG. 4. Plot of the eigenvalues β_n vs the normalized light intensity $\tilde{I}$ for the reference steady state $\phi = 0$ and optical sample length $\tilde{L} = 1.6\pi$. The instability region of the reference state is dotted.

$$\beta_0 \cong \mp 2\,\tilde{I}\ (\sin \tilde{L}/\tilde{L}),$$

$$\beta_n \cong -n^2\pi^2 \quad (n=1,2,3,\ldots), \tag{21}$$

where the upper sign applies to the state $\phi = 0$ and the lower to $\phi = \frac{1}{2}\pi$. From the first of Eqs. (21) we see that if $k\pi \leqslant \tilde{L} \leqslant (k+1)\pi$ with odd k, β_0 is positive for the state $\phi = 0$, and hence this state is unstable in the limit $\tilde{I} \to 0$ (see Fig. 4). For the same $\tilde{L}$, the state $\phi = \frac{1}{2}\pi$ is stable instead.

This may be surprising, at a first glance, because one expects that for a liquid crystal having positive dielectric anisotropy the state where the director $\hat{n}_C$ is parallel to the linearly polarized optical field should be always stable, especially if the torque at the film free surfaces is absent, as assumed in the present work.

But a simple argument shows that this may be false in the present case, because light, unlike static external fields, carries angular momentum. Suppose, in fact, that the liquid-crystal molecules, initially parallel to the optical field in the whole sample, suffer a small fluctuation in their azimuthal angle ϕ. Then, the light will become elliptically polarized in traversing the medium and the emerging beam will carry angular momentum. Since the input beam is linearly polarized, it does not carry angular momentum, on the average, so that the angular momentum carried by the output beam is also the net average angular momentum lost by the sample. Now, the sign of the angular momentum carried by the output beam depends on its polarization elicity, which, in turn, depends on the sample phase thickness $\tilde{L}$, and it may be both positive and negative. The result is that, depending on $\tilde{L}$, it may happen that the sign of the angular momentum deposited by the beam in the medium is such that the corresponding torque on the sample tends to further increase the eventual fluctuation of ϕ, leading to instability. It is evident, that the instability of the aligned state $\phi = 0$ is strictly related to the vectorial nature of the light, and that this effect could not be observed with static fields. A more careful inspection of the linearized equations confirms this connection between instability and angular momentum transfer. Finally, we remark that in the limit of a very thin film ($\tilde{L} \ll \pi$) the optical torque τ_0 in Eq. (4) reduces to the torque used by Ong in his paper. In this limit, the parallel state $\phi = 0$ is always stable for small intensity, because β_0 is positive as shown by Eqs. (21).

We conclude this section with some considerations about the effect of the lateral boundaries on the suspended film. Freely suspended smectic films although very stable usually break into several domains having different orientations. By carefully controlling the environment and temperature, single-domain suspended films can be made of several millimeters in diameter. Observations show that the anchoring at the frame holding up the film can be considered strong.[11] The initial orientation of the domain depends on small unavoidable asymmetries in the frame. The anchoring forces at the film lateral boundaries tend to maintain its primitive orientation so that a threshold is expected in the laser intensity to induce reorientation. Four lengths characterize the phenomenon: the linear dimension D of the domain, the film thickness L, the laser beam waist w_0 at the film, and the length $\bar{\lambda} = \lambda/\Delta n$ in z direction, over which the beam polarization changes appreciably. In usual experimental conditions we have $D \gg w_0 \gg L \simeq \bar{\lambda}$. The plane-wave approximation used in this work is therefore appropriate to describe the evolution of the light polarization in traversing the medium with great accuracy, but, in a more realistic model, one should take into account that the molecular reorientation is confined in a region close to illuminated region and that it could depend also on the beam profile. The effects due to the finite-beam cross section have been considered in the paper by Ong[4] by assuming a cylindrical beam profile and a weakly distorted very thin sample ($L \ll \bar{\lambda}$). When $L \simeq \bar{\lambda}$, the effects due to the polarization changes become important, as shown in the present work, but, unfortunately, an analytical calculation cannot be worked out even for a cylindrically shaped beam in the small-distortion regime. Only a rough estimate of the order of magnitude of the effects related to the finite cross section of the laser beam can be done. Due to the anchoring at the film lateral boundaries, the molecular distortion is limited to a region of diameter of about w_0 near the focus of the beam. The transverse elastic torque on the director $\hat{n}_C$ can be accounted for by a term having the form $k_t \nabla^2 \phi (\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2)$ on the left-hand side of Eq. (4). Here k_t is an effective elastic constant accounting for the stresses in the (x,y) plane. For very small distortion, we have $k_t \nabla^2 \phi \simeq -\phi/w_0^2$ (see also Ref. 3), which, for $k_t \simeq k_s$, produces in the stability analysis a constant shift in all eigenvalues β_n of about $-(\pi L/w_0)^2 \ll 1$. This shift renders the eigenvalue β_0 in Eqs. (21) negative if $\tilde{I}$ is low enough, irrespective of the actual value of $\tilde{L}$. We conclude, therefore, that the parallel state $\phi = 0$ is always stable at very low intensity when the beam has a finite cross section, because of the anchoring forces at the holding frame. But the threshold to induce reorientation may be very low if $\tilde{L}$ is between $n\pi$ and $(n+1)\pi$, with odd n. The threshold becomes negligibly small in the limit of large beams ($w_0 \to \infty$). Since in this case $I_{\text{th}} \alpha 1/w_0^2$, we have actually a

threshold P_{th} in the power of the beam. As shown in Ref. 4, for typical Sm-C materials P_{th} is of the order of a few milliwatts, which may be interesting for applications.

V. CONCLUSIONS

We have presented a theory to describe the optical reorientation induced by a linearly polarized laser beam in a freely suspended Sm-C liquid crystal. Unlike previous works on this subject, we considered the effects due to the change of the polarization as the beam propagates through the film. We found that in the plane-wave approximation the torque equation and the equations governing the change in the polarization of the beam can be solved analytically in terms of Jacobi's elliptic functions. The solution has many branches and has been discussed in detail. The stability of the trivial solutions $\phi = 0$ and $\phi = \frac{1}{2}\pi$ has been also studied. We found that the branching points of the trivial solutions do not coincide, in general, with a change of stability. Moreover, the state $\phi = 0$ aligned along the beam polarization direction, was found to be unstable in the limit of low intensity for particular ranges of values of the film thickness. Although this result may be surprising at a first glance, it can be explained physically in terms of angular momentum transfer between the radiation field and the sample. Finally, qualitative arguments show that this instability is weakly removed in the case of finite cross section of the laser beam and of strong anchoring at the frame holding the suspended film. For appropriate film thickness, however, the laser power threshold needed to induce reorientation may be very low, of the order of few milliwatts.

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