

# Research Curriculum of Fedele Lizzi

References are to my list of publications.

Starting from the early nineties my research has been mainly in the field of Noncommutative Geometry. Planck scale physics requires certainly new mathematical tools. The generalization of geometry which goes under the name of noncommutative geometry is likely to be the best tool for the job. In noncommutative geometry the focus shifts from points, lines etc. to a description based on the algebra of fields. I have been among the first to study the physical applications of noncommutative geometry (well before the great surge of interest spurred by the article of Seiberg and Witten), and have investigated phenomenological and cosmological consequences.

Before noncommutative geometry I have also worked on topological solitons (mainly skyrmions), and string theory.

A more detailed description of my research follows.

## Spectral Action, Weyl anomalies, Higgs-Dilaton

[a71, a65, a67, c15]

With A.A. Andrianov and my student M.A. Kurkov I investigated the role of Weyl symmetry and Weyl anomalies for the finite mode regularization. This is the imposition of a cutoff by considering only a finite number of eigenvalues of the (generalized) Dirac operator, and although it was devised before the noncommutative geometry approach to the standard model, it is very well suited to it. We showed that the bosonic spectral action can be inferred by the renormalization flow of the fermionic part. The presence of the dilatonic mode is also a consequence of this flow, and we calculated its action and coupling with the Higgs field.

## Noncommutative Field Theory

[a46, a56, a57, a59, a60, a61, a62, p1, p2, p3, b1, c12]

Quantum field theory on a noncommutative space is an important aspect of noncommutative geometry. It has several interesting features, among which the infrared/ultraviolet mixing, and the issues relating to quantum symmetries. With my student Galluccio and Vitale we have studied the mixing for scalar field theories with products which are alternative to the usual Grönewold-Moyal one, like the Wick-Voros product [a62] or a general translation invariant product [a61]. We found the persistence of the mixing, thus showing it is a consequence of the commutator of the coordinates (which is invariant for these theories).

As far the symmetry is concerned, this can be restored considering a quantum group,

so to have a deformed invariance. With Vitale and Aschieri [a59] I have developed a canonical procedure to twist the theory and the symmetry, so to restore the symmetry, and then we applied it to the calculation of the S-matrix for a theory in the presence of the Wick-Voros product. With Gracia-Bondía, Ruiz-Ruiz, Vaidya e Vitale I studied general properties of covariance [a57] of the product and applications to conformal field theories [a56, a60].

With Szabo and Zampini [a46] I discussed the Lie algebra structure of noncommutative gauge theory, this results an infinite dimensional version of  $SU(\infty)$ .

In a book with Aschieri, Dimitrijevic, Kulish and Wess [b1] we have reviewed the noncommutative spaces obtained by twist defroming the product.

## **Phenomenological and Cosmological Applications of Noncommutative Geometry**

[a27, a30, a32, a34, a35, a45, a48, a58]

In [a27, a45 a48] with Mangano Miele Sparano and Peloso we discussed the possible consequences of noncommuting coordinates for inflation. Papers [a27, a45] are in the context of the Connes-Lott approach to the standard model, and there we see how the Randall-Sundrum dilatonic solution of the hierarchy problem is already present in the Connes-Lott model coupled with gravity. The paper [a48] has been one of the first to consider cosmological consequences of noncommuting coordinates, setting limits to non-commutativity from the cosmic microwave background.

I have studied also, in various collaborations with Mangano, Miele, Sparano Gracia-Bondia, Varilly, Figueroa the approach pioneered by Connes to the standard model of elecroweak and strong interaction, and the spectral action. In [a32, a35] we noted the fermion doubling phenomenon, which has been recently solved by Chamseddine, Connes and Marcolli together with the problem of neutrino mass. In [a58] I discussed the connections of this new verion of the model with string theory.

## **String and Noncommutative Geometry**

[a33, a36, a37, a38, a40, a41, a42, a43, a58, c7, c8, c9]

Strings are a credible candidate to describe quantum gravity. With Landi and Szabo I was among the first to investigate the connections with noncommutative geometry, with the construction of a “noncommutative geometry of strings based on vertex operators [a37]. In [a36, a37, a40] we have seen how the  $O(d, d, \mathbb{Z})$  of target space is a gauge transformation of this noncommutative geometry. An analogous case to electromagnetism [a38]. In [a41] We have seen how the string theory at Planck energy is a generalization of the noncommutative torus.

I also studied (with Mavromatos and Szabo) the connections with branes in [a33, a39].

In [a42] I discussed, using spectral techniques mutated from noncommutative geometry, how the size of the target space is not a gauge invariant.

## **Fuzzy Spaces, Finite and Matrix Approximations**

[a24, a26, a28, a29, a31, a44, a50, a51, a52, a53, a54, a70, c5, c6, c10, c11]

Fuzzy spaces are finite approximations of spaces which maintain the symmetries of the problem, at the price of a noncommutative algebra. Con Vitale and Zampini [a50, a52, a54, c10, c11] we have introduced a fuzzy version of the Disc based on coherent states and the Wick-Voros product.

With Landi and Szabo we studied matrix approximations to the Noncommutative Torus and we have seen how this noncommutative space is recovered in the limit [a44, a51, a53].

With D'Andrea and Varilly we calculated the metric properties of the sphere, showing the the distance among coherent states converge to the usual metric distance.

With Spisso in [a69] I studied field theories on the fuzzy disc, showing how numerical simulations on it are better suited also to study phase transitions on the noncommutative plane.

## **Alternative Noncommutative products**

[a47, a49, c7, c8]

Noncommutative geometries identified by non commuting coordinates are not unique, in [a47, a49] we studied some less trivial noncommutative spaces, In [a47] (with Gracia-Bondia, Vitale e Marmo)) we proposed a class of noncommutative geometries realised by an explicit  $\star$  product coming from the reduction of integrable systems.

In [a49] (with my students Agostini e Zampini) we studied the connections between suitably generalised Weyl systems and  $\kappa$ -Minkowski spaces. With A.P. Balachandran, G. Bimonte, E. Ercolessi, G. Landi, G. Sparano e P. Teotonio we have introduced [c5, a26, a31] a finitary approximations of topological spaces, based on a noncommutative geometry of projectors and compact operators, which maintains the topological information, and in which limit [a28, a29] the original space can be recovered. In [a24] we studied Connes distance on a lattice and anomalous metric.

## **Lie-Poisson Manifolds**

[a22, a55]

These are the classical counterpart of quantum groups. In [a22] with Marmo, Sparano and Vitale I studied the dynamics on these spaces connecting them with dissipative systems. In [a55] with Stern and Vitale we studied the classical limit of noncommutative spheres seen as Lie-Poisson manifolds.

## **Propagation of Singularities**

[a23]

In [a23] (with Marmo, Sparano e Vinogradov), I studied the propagation of singularities of differential equations, seen as quasi particles, and the problem of reduction.e.

## **Highly Excited Strings and Hagedorn Transition**

[a18, a19, a20, a21, c4]

With Senda Viswanathan we studied the behaviour of strings at high density using a model based on nucleation [a18, a19, c4]. We found that the Hagedorn phase transition can be seen as a transition from a mixed a gas phase to the coalescence unto a single cluster. This has been then applied to the QCD quark-gluon plasma transition [a21].

## **Discretized Witten String Field Theory**

[a13, a14, a15, a16]

These papers in collaboration with Bordes develops computational methods to deal with Witten String Field theory. The method works very well even for a rather small number of points.

## **Null Strings and Topological Aspects of Strings**

[a8, a9, a10, a11, a17, a25]

With Balachandran Sorkin and Sparano we introduced [a8, a9, a11, c3] an alternative (phase space) Lagrangian for strings, which enabled an unified treatment of tensionful and tensionless (null) strings. The latter can be quantized [a10, a17, a25] and shown (with Rai, Sparano and Srivastava) not to have a critical dimension.

## **Four Dimensional Model of the Superstring with $N = 2$**

[a12]

In this work with D'Adda we considered the  $N = 2$  superstring. While the critical dimension of this string is 2, we show that the bosonic part of the superfield actually enables the construction of a four dimensional theory and  $N = 1$ . We also showed that the four points amplitudes of the theory vanish.

## Skymions

[a4, a5, a7, c1, c2, u3]

With Balachandran, Barducci, Gomm, Rodgers and Stern we worked on skymions, and in particular on the three flavours model. We found the presence of the dibaryon  $H$  [a4, a5] in the theory and studied its phenomenology, as well as the case of higher baryon number [a7]. The mass of the  $H$  is found very close to the  $\Lambda\Lambda$  resonance, in accordance with the analogous QCD calculations.

## Hadron and String Spectroscopy

[a6]

I analyzed with Rosenzweig [a6] the spectrum of mesons with one heavy and one light quark in the bag and string models, finding that the deviations from the ideal Regge trajectory can be explained by the presence of massive quarks at the end of the string/flux tube. The predictions we made, with the crude technology of the time, are in excellent agreement with the finding of the Argus collaboration.

## Monopoles and other Topological Solitons

[a1, a2, u2, a3, u1, u2]

Apart from skymions I have also worked on other topological solitons, with Balachandran, Rodgers and Nair. I studied monopoles and their confinement in the hadronic bag model [a1]. We also studied non abelian monopoles in unified theories [u2], self-adjointness of instantons and merons [a2], and condensed matters of nonabelian monopoles:  ${}^3\text{He}$  vortices and biaxial nematics, showing how these nonabelian defects reduce global symmetries.

Also my undergraduate thesis [u1] was on the fibre bundles formalization of monopoles and the Aharonov-Bohm effect.