

THE CHEMICAL COMPOSITION OF WHITE DWARFS AS A TEST OF CONVECTIVE EFFICIENCY DURING CORE HELIUM BURNING

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ABSTRACT

Pulsating white dwarfs provide constraints to the evolution of progenitor stars. We revise He-burning stellar models, with particular attention to core convection and to its connection with the nuclear reactions powering energy generation and chemical evolution. Theoretical results are compared to the available measurements for the variable white dwarf GD 358, which indicate a rather large abundance of central oxygen (Metcalf and coworkers). We show that the attempt to constrain the relevant nuclear reaction rate by means of the white dwarf composition is faced with a large degree of uncertainty related to evaluating the efficiency of convection-induced mixing. By combining the uncertainty of the convection theory with the error on the relevant reaction rate, we derive that the present theoretical prediction for the central oxygen mass fraction in white dwarfs varies between 0.3 and 0.9. Unlike previous claims, we find that models taking into account semiconvection and a moderate $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate are able to account for a high central oxygen abundance. The rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ used in these models agrees with the one recently obtained in laboratory experiments by Kunz and coworkers. On the other hand, when semiconvection is inhibited, as in the case of classical models (bare Schwarzschild criterion) or in models with mechanical overshoot, an extremely high rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction is needed to account for a large oxygen production. Finally, we show that the apparent discrepancy between our result and those reported in previous studies depends on the method used to avoid the convective runaways (the so-called breathing pulses) that are usually encountered in modeling late stage of core He-burning phase.

Subject headings: convection — nuclear reactions, nucleosynthesis, abundances — stars: evolution — stars: interiors — white dwarfs

1. INTRODUCTION

White dwarfs (WDs) are dead stars that supply the energy irradiated from the surface by consuming their thermal reservoir. Since the thermal content of a WD depends on its chemical composition (see Van Horn 1971), evaluation of the cooling timescale requires a good knowledge of the processes that modify the original composition of the progenitor. In addition, some observed features of Type Ia supernova outburst depend on the detailed internal composition of the exploding WD (Domínguez, Höflich, & Straniero 2001). According to the theory of stellar evolution, the majority of WDs are post-asymptotic giant branch (AGB) stars (Paczynski 1970a). In such a case, they would consist of the primary ashes of He burning, essentially carbon and oxygen: C is initially produced by the 3α reactions and, subsequently, O is synthesized via the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$.

During their long cooling time, white dwarfs cross the instability strip and undergo stable pulsations. These pulsations can be observed as variations in brightness that could be used to provide constraints on the internal structure of these condensed objects. This is an important tool to test the reliability of progenitor models. Metcalf, Nather, &

Winget (2000) analyzed seismic data of the variable GD 358, a DB-type white dwarf, and estimated the WD mass, the effective temperature, and the mass of the thin helium-rich envelope. This method was recently extended to provide a description of the internal chemical profile (Metcalf, Winget, & Charbonneau 2001; see also Metcalf, Salaris, & Winget 2002). In particular, a large oxygen abundance was found in the innermost region, namely, 0.84 ± 0.03 . By comparing this chemical profile with those predicted by theoretical models of WD progenitors, it was concluded that the reaction rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ is definitely larger than those recently derived in laboratory experiments (Buchman 1996; Kunz et al. 2002).

We believe that these constraints on the rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction are model dependent. We will show how the predicted WD composition changes when the scheme for the convective mixing is changed. The connection between nuclear reaction and convection and their influence on the final amounts of C and O in the He-exhausted cores of stars with a mass between 0.8 and $25 M_{\odot}$ have been recently revised by Imbriani et al. (2001). In particular, they showed that oxygen production via $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ may be substantially increased by extending

the central mixing during the late stage of the core He-burning phase, when the central He mass fraction falls below about 0.1.

Unfortunately, in spite of the many theoretical works published over the last three decades, the physics that determines the extent of the convective region within the He core is still poorly known. The theoretical calculations available so far leave various scenarios open. Classical models, those based on a bare Schwarzschild criterion (as presented by Iben & Rood 1970), are still calculated and widely used in many studies (e.g., Umeda et al. 1999; Althaus et al. 2002). Nonetheless, models that include some algorithm to handle the discontinuity of the opacity that forms at the external border of the convective core as a consequence of the conversion of He into C (and O) should be considered as more reliable (Pacziński 1970b; Castellani, Giannone, & Renzini 1971a, 1971b; Demarque & Mengel 1972; Robertson & Faulkner 1972; Sweigart & Demarque 1972; Sweigart & Gross 1976; Castellani et al. 1985; Iben 1986; Sweigart 1990; Lattanzio 1991; Dorman & Rood 1993; Domínguez et al. 1999 and references therein). We remind that this phenomenon naturally leads to the growth of the convective core (the so-called induced overshoot) and to the formation of a semiconvective layer outside the fully convective region. Alternative models assume that a mechanical overshoot (i.e., due to the inertia of the material accelerated by the buoyancy forces) takes place at the boundary of the convective region (Saslaw & Schwarzschild 1965; Shaviv & Salpeter 1973; Maeder 1975; Bertelli, Bressan, & Chiosi 1985; Bertelli et al. 1990; Shaller et al. 1992; Bressan et al. 1993; Herwig et al. 1997; Girardi et al. 2000, and references therein). The existence of this phenomenon cannot be questioned in the framework of a reliable physical scenario. However, the quantitative relevance of the mechanical overshoot is a matter of a hard debate (Renzini 1987). The most recent attempts to calibrate the extension of the convective overshoot for centrally H-burning stars conclude that it should be confined within $(0.2-0.3)H_p$, where H_p is the pressure scale height (Stothers & Chin 1992; Bressan et al. 1993; Demarque, Sarajedini, & Guo 1994; Mermilliod, Huetamendia, & del Rio 1994; Schröder, Pols, & Eggleton 1997; Testa et al. 1999). Concerning the core He-burning phase, a moderate mechanical overshoot mimics the effect of the induced overshoot. On the contrary, a large mechanical overshoot (say $1H_p$ or larger) would cancel out the semiconvective zone, and major changes in the theoretical scenario are expected.

Since the theoretical uncertainty does not provide a satisfactory answer to the problem of stellar convection, some observational constraints have been investigated. In particular, the luminosity function of bright globular cluster stars could be used to provide some indications on the correct mixing scheme. The larger the mixing during the core He-burning, the less He fuel will be left for the subsequent AGB evolutionary phase. Thus, the ratio R_2 (number of stars observed in the AGB/number of stars observed in the horizontal branch) may be used to constrain the mixing efficiency (see, e.g., the discussion in Renzini & Fusi Pecci 1988). The measured values of R_2 in globular clusters (0.15 ± 0.01 ; Buzzoni et al. 1983; Buonanno et al. 1985) support semiconvective models. An alternative method, based on red giant branch stars, has been proposed by Caputo et al. (1989); their conclusions agree with those based on the R_2 parameter.

The knowledge of the internal composition of WDs provides an independent method to constrain the physics controlling chemical variations in the core of He-burning stars. In this context, the result obtained by Metcalfe and coworkers appears in contrast with the indications derived from the globular cluster luminosity functions. In fact, they showed that semiconvective models cannot reproduce the large value of the oxygen measured in GD 358. As a possible solution to this problem, they propose a substantial enhancement of the rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction with respect to the experimental values. In this paper, we review various convective schemes and discuss their impact on the predicted core composition. Contradicting previous claims, we will show that fully semiconvective models can account for a relatively high value of the central oxygen, even if a moderate rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction is used, in agreement with the most recent laboratory measurements.

2. FIVE MODELS FOR THE CORE He BURNING PHASE

In this section, we revise the theoretical expectations for core He-burning models. The stellar structure at the beginning of the core He-burning phase has been obtained by evolving a $3 M_\odot$ model with solar composition ($Z = 0.02$, $Y = 0.28$) since the pre-main sequence. Then, five He-burning models have been calculated under different assumptions for the convective scheme. We have selected the mixing algorithms by searching in the recent literature for the models more representative of the theoretical scenarios typically adopted for He-burning stars. The choice for the initial stellar parameters is adequate for the progenitor of a disk WD with mass of about $0.65 M_\odot$, which is the mass estimated by Metcalfe, Winget, & Charbonneau (2001) for the variable WD GD 358. Variations of the initial mass and chemical composition do not significantly affect the main conclusion of the present work. In any case, a detailed description of the internal chemical stratification in WDs generated by progenitors with masses ranging between 1 and $7 M_\odot$ and metallicity in the range $Z = 0-0.02$ has been reported by Domínguez, Höflich, & Straniero (2001). All five models were obtained by using the rate provided by Kunz et al. (2002) for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction. The value of this rate is about a factor of 2 lower than the one claimed by Metcalfe, Salaris, & Winget (2002) to explain the observed composition of GD 358. Some relevant properties of the computed models are reported in Table 1.

2.1. Bare Schwarzschild Models

In the case of classical models (Iben & Rood 1970), the extension of the convective core is simply determined by the condition $\nabla_{\text{rad}} > \nabla_{\text{ad}}$ (the so-called Schwarzschild criterion). No special algorithms to account for the induced overshoot and semiconvection are considered. We are well aware that this kind of He-burning model is out of date. Nevertheless, they are still used, particularly in studies of stellar progenitors of WDs (Althaus et al. 2002) and of Type Ia supernovae (Umeda et al. 1999).

Since the bare Schwarzschild criterion provides only a lower limit to the size of the convective core, the resulting He-burning lifetime is particularly short and the final central oxygen abundance is definitely lower than the one derived from seismic data of variable WDs (see Table 1).

TABLE 1
CORE HE-BURNING MODELS: $M = 3 M_{\odot}$, $Z = 0.02$

Label (1)	f^a (2)	τ_{He}^b (3)	X_{C}^c (4)	X_{O}^c (5)	M_D^d (6)
BSM	1	88	0.42	0.56	...
SM	1	145	0.19	0.79	0.31
PSM	1	134	0.40	0.58	0.27
HOM	1	153	0.42	0.56	0.32
LOM	1	139	0.38	0.60	0.28
SM	0.4	135	0.52	0.46	0.29
SM	1.6	149	0.08	0.90	0.31
HOM	0.4	142	0.66	0.32	0.31
HOM	1.6	157	0.28	0.70	0.32

^a Enhancement factor of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate. The quantity $f = 1$ corresponds to the Kunz et al. 2002 rate.

^b He-burning lifetime (Myr).

^c Final central mass fractions of C and O.

^d Location, in $M_{\odot}$, of the sharp discontinuity that marks the separation between the innermost low-C region and the external zone unchanged by the central convective episodes experienced by the star during core He-burning phase (see Fig. 4)

The evolution of the central composition for this model (BSM, bare Schwarzschild model) is shown in Figure 1a. On the other hand, a large fraction of He remains unburned and a longer AGB phase takes place. Then, the predicted value for the R_2 parameter is rather large (~ 0.7), in clear disagreement with the value obtained for globular cluster stars.

2.2. Semiconvective Models

In this case, we have used a numerical algorithm to handle the induced overshoot and the consequent semiconvection (see Castellani et al. 1985). Starting from the center of the star, a small mass fraction (namely a mesh point) is added to the top of the convective core. Then, this procedure is iterated until convective neutrality is achieved at the external border (convective core) or inside the well-mixed region (convective core plus semiconvective zone). In the latter case, a detached convective shell forms, whose external border is moved outward until $\nabla_{\text{rad}} = \nabla_{\text{ad}}$. The resulting evolution of the internal He profile is shown in Figure 2 (*left panel*). Note the ongoing growth of the fully convective core (the most internal and flat zone) and the formation of the outermost partially mixed region (semiconvective zone).

As it is well known, when the central He is substantially depleted, some instabilities (named breathing pulses) take place at the external border of the convective core. In this phase, even a small ingestion of fresh He causes a significant increase of the nuclear energy production, thus leading to a convective runaway. On the basis of both theoretical and observational constraints, breathing pulses are usually attributed to the adopted algorithms rather than to the physics of convection. Following the suggestion of Dorman & Rood (1993), we have suppressed these instabilities by setting to zero the gravitational (thermal) energy in the core when the central mass fraction of He drops below 0.1. Note that in this phase, the gravitational energy generally accounts for a small positive contribution to the total energy (less than 1%). Only during a breathing pulse, to counterbalance the sudden increase in the thermonuclear energy flux, does the contribution of the gravitational energy become large and negative. By neglecting this term in the energy bal-

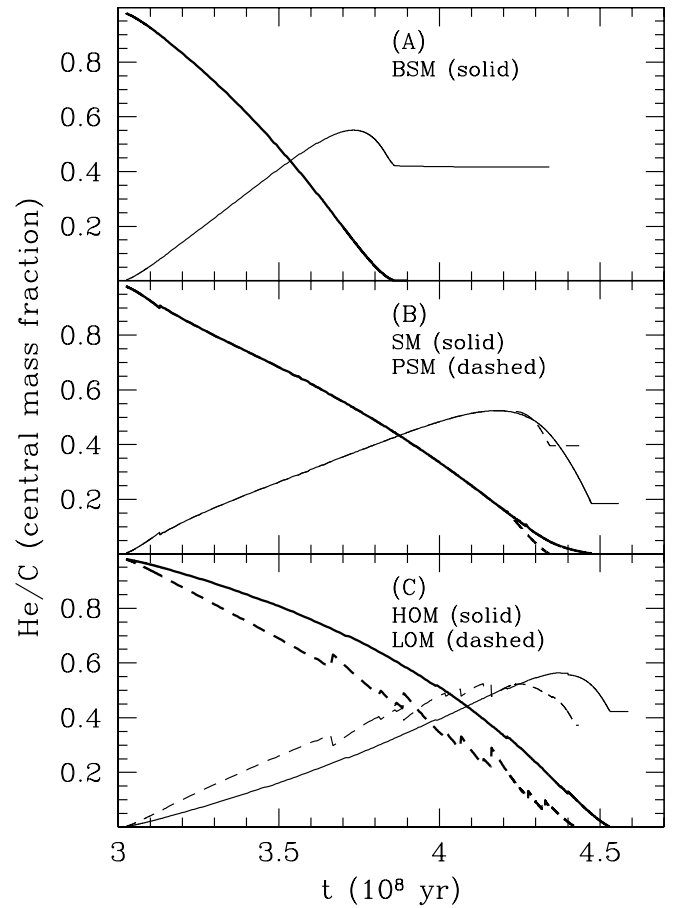


FIG. 1.—Evolution of the central He mass fraction (*bold line*) and that of the central C mass fraction (*thin line*) as obtained by changing the treatment of the core convection: BSM = bare Schwarzschild model (no overshoot, no semiconvection), SM = semiconvective model (breathing pulse suppression as in Dorman & Rood 1993), PSM = semiconvective model (breathing pulse suppression as in Caputo et al. 1989), HOM = high overshoot model ($1 H_p$), LOM = low overshoot model ($0.2 H_p$).

ance equation, the external border of the mixed region slowly recedes and breathing pulses are avoided. This case is reported in Table 1 with the label SM (semiconvective model). The corresponding evolutions of the central abundances of He and C are reported in Figure 1b (*solid lines*). Note the evident decrease of the He consumption rate in the final part of the core He-burning phase. This is due to the efficiency of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction that releases almost the same energy as the 3α with one-third of the fuel consumption. This delay of the final part of the He burning has a great influence on the final core composition: a large oxygen mass fraction (0.79) is left at the center. This value is only slightly lower than that found for GD 358. We have already recalled that semiconvective models also provide the best reproduction of the R_2 ratio observed in globular clusters.

Obviously, the artificial suppression of the breathing pulses may be obtained by means of different methods. In principle, there are no evident reasons to prefer one method with respect to another. A rather diffused algorithm has been proposed by Caputo et al. (1989). The growth of the convective core is limited by the constraint that the central He abundance can not increase with time, when the central He mass fraction drops below 0.1. In practice, this method

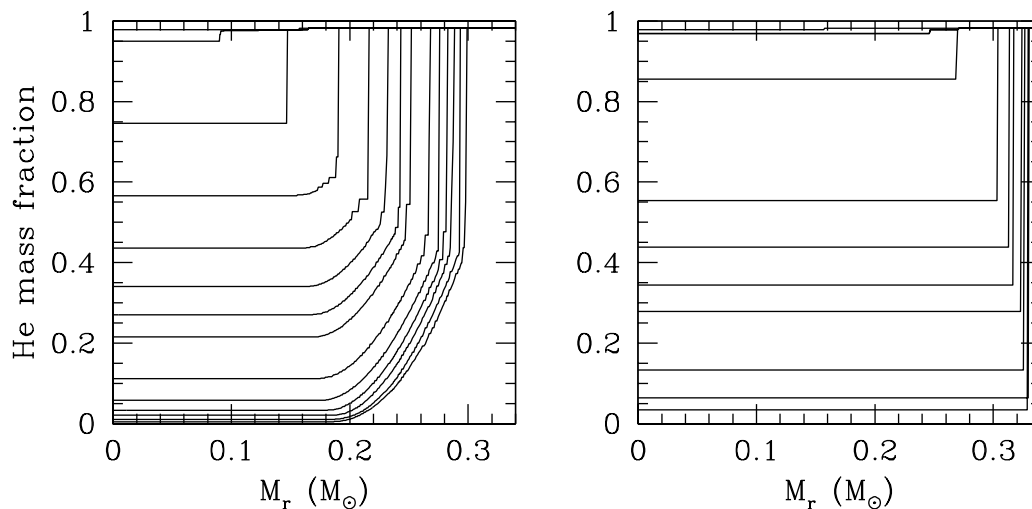


FIG. 2.—Evolution of the internal He profile for the semiconvective model (SM; *left panel*) and for the high overshoot model (HOM; *right panel*)

strongly reduces the effects of the induced overshoot and semiconvection during the late stage of core He burning. As a result, the He-exhaustion phase of this model is similar to the one found in the case of classical models. The evolutions of the central mass fractions of He and C for this model are shown in Figure 1*b* (*dashed lines*). This model is reported in Table 1 with the label PSM (partially semiconvective models). As in the case of SM, PSM provides a quite good reproduction of the observed R_2 ratio. However, since the C depletion essentially takes place during the late stage of the core He burning, PSM predicts a moderate oxygen production very similar to the one obtained in the BSM case. Note that PSM is the model adopted by Metcalfe, Winget, & Charbonneau (2001) in their analysis of the chemical composition of GD 358.

2.3. Mechanical Overshoot

An alternative scenario could arise if a sizeable mechanical overshoot induces an efficient mixing of the region located beyond the external border of the fully convective core. In this case, no special algorithms are used to handle semiconvection or breathing pulses (Girardi et al. 2000). The case of a moderate overshoot, namely $0.2H_p$ (LOM, low overshoot model), is illustrated in Figure 3, where we show the radiative and the adiabatic temperature gradients for three different epochs, namely, when the residual mass fraction of He at the center is 0.76, 0.44, and 0.2, in panels (a), (b) and (c), respectively. At the beginning of the He burning (Fig. 3*a*), mechanical overshoot brings the discontinuity of the radiative gradient (*solid line*) below the level of the adiabatic gradient (*dashed line*) so that the external border of the convective core is stable. It goes without saying that mechanical overshoot mimics the job done by the induced overshoot in semiconvective models. However, as shown in the other two panels, when He burning goes ahead, a convective shell forms with an unstable external boundary. A partial mixing beyond this boundary (semiconvection) or some form of overshoot, capable of restoring the convective neutrality, is obviously required. The resulting evolutions of the central He and C are reported in Figure 1*c*. When the central He mass fraction drops below 0.6, some rapid variations of the core composition take place.

Their origin is easily understood by looking at the sequence reported in Figure 3. Owing to the overshoot, the size of the convective core increases until its external boundary reaches the base of the previous convective shell and, in turn, a substantial amount of fresh He is suddenly ingested.

The inconsistency of such kind of models may be removed if a moderate overshoot is also applied to the exter-

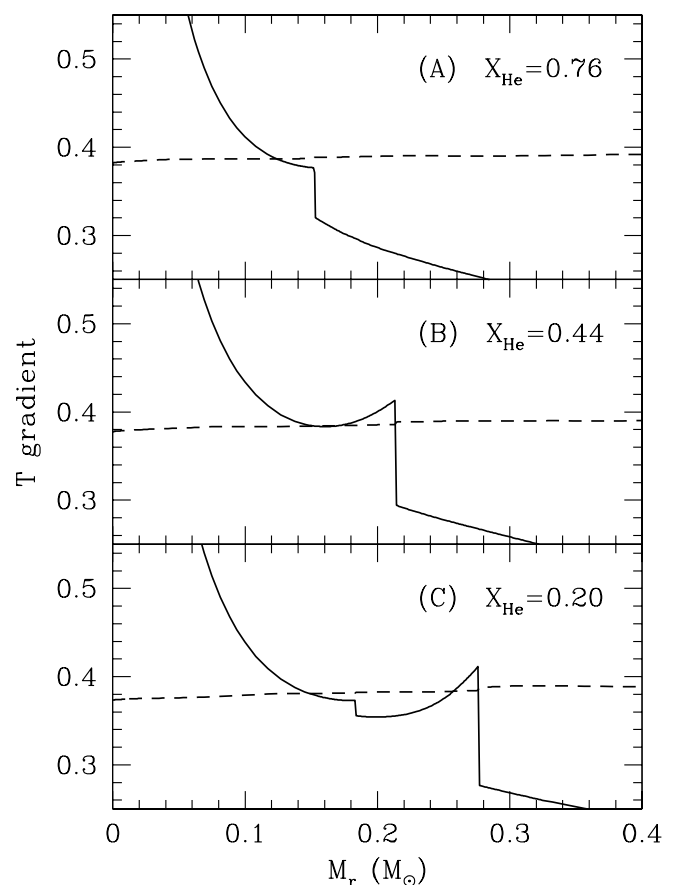


FIG. 3.—Radiative (*solid line*) and adiabatic (*dashed line*) temperature gradients in the core of the LOM model ($0.2H_p$ overshoot model). X_{He} is the central mass fraction of He.

nal border of the convective shell. In this case, the resulting final composition would be very similar to the one obtained in the case of a semiconvective model. On the contrary, a significantly different scenario arises if a large mechanical overshoot takes place at the external border of the convective core. In this case, the semiconvective zone would be swept away. We have calculated an additional model by applying an overshoot of $1H_p$ (HOM, high overshoot model). Under this assumption, the mass of the material within the classical border is about half of the mass actually mixed. As the He burning proceeds, the mixing rapidly extends up to about $0.3 M_\odot$ so that any trace of semiconvection is canceled out (see Fig. 2, *right panel*) and no convective shells form. The evolution of the central composition is shown in Figure 1c (*solid lines*).

It should be noted that overshoot models are not immune to breathing pulses. For this reason, LOM and HOM have been calculated by adopting the same method used in SM to avoid these instabilities. A second clarification regards the fact that no overshoot has been considered for the computation of the H-burning phase. As it is well known, an overshoot occurring during this phase would produce a larger He core mass, which is equivalent to the case of a larger stellar mass without overshoot. However, as we have already shown, a change of the stellar mass does not substantially modify the final central amounts of C and O.

In summary, the late stage of the core He burning in overshoot models is particularly fast. As a consequence, a smaller amount of oxygen is produced with respect to fully semiconvective models. However, if the overshoot is small enough, a semiconvective layer survives, thus increasing the oxygen production. For these models, a suitable reproduction of the R_2 ratio may be obtained by tuning the size of the overshoot zone. However, this is a calibration of the model rather than a prediction. In this framework, a large overshoot is ruled out because it would imply too low a value of the R_2 parameter (Renzini & Fusi Pecci 1988).

2.4. Time-dependent Convection

All the models previously described have been obtained, as usual, by assuming an instantaneous mixing within the convective core and, eventually, in the overshoot region. In the last few years, some papers report calculations of AGB models obtained by adopting a time-dependent mixing scheme (Herwig et al. 1997; Mazzitelli, D'Antona, & Ventura 1999; Cristallo et al. 2001; Chieffi et al. 2001). Note, however, that since the convective velocities are evaluated in the framework of the mixing length theory, the resulting mixing within the convective regions is very efficient, practically instantaneous. Nevertheless, the relevant point for the present work is that during the TP-AGB phase, the convective envelope penetrates the He-rich intershell (the so-called third dredge-up). Since the opacity at the base of the convective envelope (H-rich) is larger than that found in the layer located just below (He-rich), a discontinuity in the radiative gradient takes place. This is exactly the same phenomenon that occurs at the external border of the convective core during the central He-burning phase. Indeed, such a discontinuity should induce a further mixing below the formally unstable region, namely the one defined by the bare Schwarzschild criterion. Herwig et al. (1997) propose to add an extra mixing (or overshoot) outside the boundaries of the formally convective regions. In the overshoot zones, they

assume an exponential decay of the convective velocity, in agreement with hydrodynamical simulations of shallow stellar surface convective zone (Freytag et al. 1996). A similar decay of the convective velocity has been also found by Asida & Arnett (2000) in hydrodynamical calculations of convective regions driven by an oxygen-burning shell in massive stars. Then, in order to check the effect of a time-dependent mixing on our result, we have tentatively computed a model of a core He-burning star by assuming an exponential decay of the convective velocity beyond the boundaries of the formally convective zones (see Chieffi et al. 2001 for details on the numerical algorithm). In such a way, a partial mixing is found above the fully convective core and, eventually, above the convective shell. The depth of the partially mixed regions is modulated by the difference between the radiative and the adiabatic gradient at the formally convective boundary but depends on the adopted strength of the velocity exponential decay, which is free parameter. In any case, the overall properties of the model and, in particular, the final oxygen abundance are similar to those obtained in the case of semiconvective models, thus confirming the previous investigation reported by Sweigart (1990).

3. DISCUSSION AND CONCLUSION

Table 1 summarizes the results of the five models described in the previous section. Some internal profiles of C and O at the beginning of the thermally pulsing AGB phase are shown in Figure 4. The innermost region keeps track of the mixing experienced during the core He-burning phase. In semiconvective models as well as in the model with mechanical overshoot, a sharp variation of the chemical composition around $M_r \sim 0.3 M_\odot$ separates the region that is partially or totally mixed during the core burning from the external layers, whose composition remains unchanged¹ up to the beginning of the double shell-burning phase (early AGB). In the case of a large overshoot (HOM, Fig. 4c), this sharp discontinuity coincides with the maximum extension attained by the convective core (including the overshoot). Therefore, its location depends on the assumed value of the overshoot parameter. In semiconvective models (Fig. 4b), two different regions may be distinguished below the chemical discontinuity: a central homogeneous region created by the fully convective core and an intermediate region, which coincides with the semiconvective zone. A bump in the oxygen distribution characterizes this intermediate region. Owing to the partial mixing induced by semiconvection, a small amount of He still survives in this zone at the epoch of central He exhaustion. Therefore, at the beginning of the early AGB, the temperature rises and the residual He is rapidly consumed, mainly through the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction, so that a considerable amount of O is accumulated. Salaris et al. (1997) suggested that this bump in the oxygen profile is smeared off by a Rayleigh-Taylor instability. In principle, the sharp discontinuity at $M_r \sim 0.3 M_\odot$ could be smoothed by the chemical diffusion operating during the long cooling timescale. However, minor effects are expected only in the case of bright variable WDs. In such a case, the location of the chemical discontinuity might be derived from seismic data of pulsating WDs. The last column of Table 1 shows the expected location of this discontinuity (M_D) for semi-

¹ Namely, as fixed by the first dredge-up.

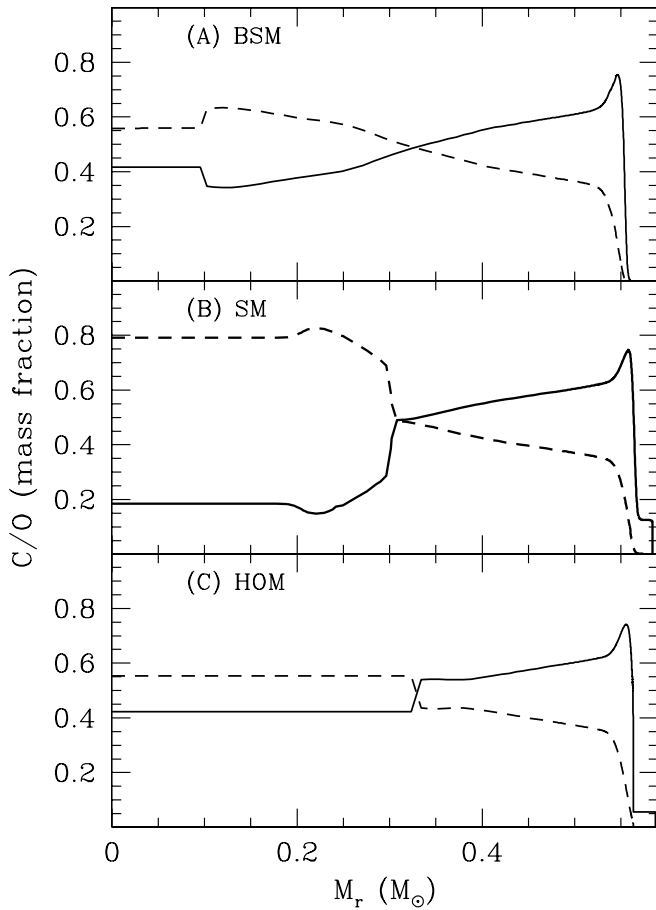


FIG. 4.—Chemical stratification, C (solid line) and O (dashed line), of the core at the onset of the thermally pulsing AGB phase.

convective and overshoot models. This may be compared with the value of the q parameter reported by Metcalfe, Winget, & Charbonneau (2001) for GD 358. According to these authors, q is the ratio between the innermost, almost homogeneous region and the total mass of the WD. Metcalfe et al. (2001) report $q = 0.49 \pm 0.01$, which would imply $M_D = 0.29$ and $0.32 M_\odot$ for $M_{WD} = 0.6$ and $0.65 M_\odot$, respectively. An inspection of Table 1 shows that the calculated values are in very good agreement with the measured quantity. As noted by Metcalfe, Salaris, & Winget (2002), since the location of M_D depends on the value of the overshoot parameter, its measure could be used to test the efficiency of the mechanical overshoot. We merely comment that this may be done only if overshoot is particularly strong ($\sim 1H_p$ or larger), otherwise M_D will be, in any case, fixed by the extension of the semiconvective region, as discussed in § 2.3.

In the case of the classical model (Fig. 4a), the innermost flat region coincides with the maximum extension of the convective core during core He burning. In this model, the small convective core does not cover the whole region where nuclear burning is efficient. Toward the end of the core burning phase, the He depletion takes place well outside the convective core. Later on, at the beginning of the early AGB, the incoming He-burning shell terminates the task initiated during the previous phase. The maximum in the oxygen profile coincides with the base of the radiative region, where only a small amount of He survives at the end of the core He burning.

The second, more stringent comparison between theory and asteroseismology concerns the central oxygen abundance. The predicted values are in column (4) of Table 1. The largest oxygen abundance is obtained in the case of the SM. The calculated mass fraction does not differ greatly from the value derived from GD 358 by Metcalfe et al. (2001) and is probably well within the error bars of the seismic data. This result has been obtained by using a moderate rate for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction, as recently reported by Kunz et al. (2002). All the other models lead to lower abundances for the central oxygen. As already discussed by Metcalfe et al. (2002), a very large rate of the carbon destruction would be required to reconcile these models with the measured central oxygen abundance in GD 358.

The evaluation of the experimental error affecting the measurements of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate is a hard task. A complete bibliography of the published experimental reports may be found in the paper of Kunz et al. (2002). The laboratory experiments have been extended down to about 1 MeV. Below this energy, the extremely small value of the cross section (<10 pb) hampers direct detection of γ -rays, and extrapolation procedures have to be used to extract the astrophysical S -factor at the energy relevant for core He burning (~ 300 keV). Such an extrapolation is based on the fitting of differential cross sections in the investigated region and requires the inclusion of the phase correlation between the two incoming partial waves that contribute to the two multipoles. A global analysis that attempts to take into account all the possible sources of uncertainty has been reported by Buchmann (1996, 1997). According to this work, at $T = 1.8 \times 10^8$ K (about 300 keV), the possible value of the reaction rate ranges between $N_A \langle \sigma, v \rangle = 0.5$ and $2.2 (10^{-15} \text{ cm}^3 \text{ mol}^{-1} \text{ s}^{-1})$. A recent compilation by NACRE (Angulo et al. 1999) adopts a slightly smaller range of accepted values, namely between 0.9 and $2.1 (10^{-15} \text{ cm}^3 \text{ mol}^{-1} \text{ s}^{-1})$. Finally, the latest experimental investigation by Kunz et al. (2002) reports $N_A \langle \sigma, v \rangle = 1.25 (10^{-15} \text{ cm}^3 \text{ mol}^{-1} \text{ s}^{-1}) \pm 30\%$.

We have investigated the influence of this uncertainty on our conclusions. Some additional models have been obtained by multiplying the rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ by a factor of $f = 0.4$ and 1.6 . This corresponds to an error bar that is double the one quoted by Kunz et al. (2002) and similar to the one accepted by Buchmann (1996). The resulting central oxygen abundances for SM and HOM are reported in the last four rows of Table 1. Even by considering such a large error, the maximum amount of central oxygen in models without semiconvection is definitely lower than that claimed by Metcalfe et al. (2001) for GD 358.

In summary, by combining the uncertainty on the relevant nuclear reaction rate and that due to convective efficiency, the theoretical predictions on the final central oxygen mass fraction range between 0.3 and 0.9. The large value reported by Metcalfe et al. favors semiconvective models. However, a moderate mechanical overshoot, which leaves unchanged the semiconvective layer, is still possible.

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