Averaging procedure in variable-G cosmologies

Vincenzo F. Cardone (Dip. Fisica, Torino) and Giampiero Esposito (INFN, Naples)

Coarse-Grained Cosmology, GGI, January 26-29, 2009

Running-G cosmologies

The renormalization-group improvement consists in the modified Einstein equations

$$(1) \qquad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda(k) g_{\mu\nu} = 8\pi G(k) T_{\mu\nu},$$

where the Newton parameter G and cosmological term Λ are now dependent on the scale k, which is the running cut-off of the renormalization group equation.

Brans–Dicke improvement

Following Reuter and Weyer, we consider a so-called Brans-Dicke approach, where G and Λ play the role of externally prescribed background fields, while we borrow from Buchert an irrotational fluid motion with Gaussian normal coordinates comoving with the fluid.

Brans–Dicke improvement

Hence we start from the field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G \rho u_\mu u_\nu + \Phi_{\mu\nu}, \tag{2}$$

where

$$\begin{aligned} \Phi_{\mu\nu} &\equiv G^{-2} \Big\{ \frac{1}{2} G_{;\mu} G_{;\nu} - G G_{;\nu\mu} + g_{\mu\nu} \Big[-\frac{5}{4} G_{;\rho} G^{;\rho} \\ &+ G \Box G \Big] \Big\}, \end{aligned} \tag{3}$$

Buchert averages

and hence contract with the vector $u^\mu \equiv \frac{\partial x^\mu}{\partial t}$ to get the scalar functions for which the Buchert averages

$$\langle\psi(t,X^i)\rangle_D \equiv \frac{\int_D \psi(t,X^i)\sqrt{\det g_{ij}}d^3X}{\int_D \sqrt{\det g_{ij}}d^3X} \quad (4)$$

are defined. From a dimensionless effective scale factor $a_D(t)$ one obtains the averaged expansion rate $\langle\theta\rangle_D = 3\frac{\dot{a}_D}{a_D}$ and backreaction term $Q_D = \frac{2}{3}\langle(\theta - \langle\theta\rangle_D)^2\rangle_D - 2\langle\sigma^2\rangle_D$.

Averaged equations with variable G

The averaged Hamiltonian constraint and averaged Raychaudhuri equation read as

$$3H_D^2 = \langle \Lambda \rangle_D + 8\pi \langle G \rangle_D \rho_{\text{eff}}^D, \tag{5}$$

$$3\frac{\ddot{a}_D}{a_D} = \langle \Lambda \rangle_D - 4\pi \langle G \rangle_D (\rho_{\text{eff}}^D + 3p_{\text{eff}}^D), \tag{6}$$

Basic definitions
where, having defined $\psi_G \equiv \log(G)$ and

$$\begin{aligned} F_1 &= \theta\psi_{G,0} - \frac{3}{4}\psi_{G,0}^2 + \frac{5}{4}h(\mathrm{grad}\psi_G, \mathrm{grad}\psi_G) \\ &- \frac{\triangle G}{G}, \end{aligned} \tag{7}$$

Basic definitions

$$F_2 = \frac{1}{3}\theta\psi_{G,0} - \psi_{G,0}^2 + \frac{G_{,00}}{G} + \frac{2}{3}h(\mathrm{grad}\psi_G, \mathrm{grad}\psi_G)$$

$$(8) - \frac{1}{3}\frac{\triangle G}{G},$$

Effective density and pressure

one has

$$\langle G\rangle_D \rho^D_{\rm eff} = \langle G\rho\rangle_D - \frac{1}{16\pi}\langle {}^{(3)}R\rangle_D - \frac{1}{16\pi}Q_D$$

$$(9) \quad + \quad \frac{\langle F_1\rangle_D}{8\pi},$$

$$\langle G\rangle_D p^D_{\rm eff} = \frac{1}{48\pi}\langle {}^{(3)}R\rangle_D - \frac{1}{16\pi}Q_D - \frac{1}{24\pi}\langle (F_1 + 3F_2)\rangle_D .$$

(10)

Cosmic quintet

We can now define the density parameters

(11) $$\Omega_m^D \equiv \frac{8\pi \langle G\rho \rangle_D}{3H_D^2}, \; \Omega_\Lambda^D \equiv \frac{\langle \Lambda \rangle_D}{3H_D^2},$$

(12) $$\Omega_R^D \equiv -\frac{\langle {}^{(3)}R \rangle_D}{6H_D^2}, \; \Omega_Q^D \equiv -\frac{Q_D}{6H_D^2},$$

Cosmic quintet

(13) $$\Omega_G^D \equiv \frac{\langle F_1 \rangle_D}{3H_D^2}.$$

The averaged Hamiltonian constraint becomes therefore

(14) $$\Omega_m^D + \Omega_\Lambda^D + \Omega_R^D + \Omega_Q^D + \Omega_G^D = 1.$$

Three effective fluids
We also introduce three auxiliary effective fluids with energy densities

$$(15)\rho_M^D \equiv \frac{\langle G\rho\rangle_D}{\langle G\rangle_D} - \frac{3\langle F_2\rangle_D}{8\pi\langle G\rangle_D}, \rho_Q^D \equiv -\frac{1}{16\pi}\frac{Q_D}{\langle G\rangle_D},$$

$$(16)\quad \rho_R^D \equiv -\frac{1}{16\pi}\frac{\langle {}^{(3)}R\rangle_D}{\langle G\rangle_D} + \frac{1}{8\pi}\frac{\langle (F_1 + 3F_2)\rangle_D}{\langle G\rangle_D},$$

Three effective fluids
and effective pressure

(17) $$p_M^D = 0,\ p_Q^D \equiv -\frac{1}{16\pi}\frac{Q_D}{\langle G\rangle_D},$$

(18) $$p_R^D \equiv \frac{1}{48\pi}\frac{\langle {}^{(3)}R\rangle_D}{\langle G\rangle_D} - \frac{1}{24\pi}\frac{\langle (F_1 + 3F_2)\rangle_D}{\langle G\rangle_D}.$$

Sum rules
Interestingly, one finds

$$\rho^D_{\text{eff}} = \rho^D_M + \rho^D_Q + \rho^D_R, \tag{19}$$

$$p^D_{\text{eff}} = p^D_M + p^D_Q + p^D_R. \tag{20}$$

Physical interpretation

Our ρ_M^D acts like a matter term, whereas ρ_Q^D behaves as stiff matter. If the backreaction Q_D is positive, the pressure p_Q^D is negative, and hence our ρ_Q^D acts as a variable cosmological term. In our **arXiv:0805.1203**, we have solved for ρ_Q^D and ρ_R^D from the sum rules (19), (20).

Large-z behaviour
If, at large z, one can write

$$w_{\rm eff}^{D}(z) \equiv \frac{p_{\rm eff}^{D}(z)}{\rho_{\rm eff}^{D}(z)} \approx 0, \tag{21}$$

$$3H_D^2(z) - \langle \Lambda \rangle_D(z) \approx 8\pi G_N \rho_M^{\rm FLRW}(z), \tag{22}$$

Large z

the resulting ρ_Q^D and ρ_R^D approach 0. Thus, the dust FLRW case is recovered in the early universe. In the early universe, only ρ_M^D survives, and at late epochs it receives the new contribution $-\frac{3}{8\pi}\frac{\langle F_2\rangle_D}{\langle G\rangle_D}$.

Baryons and dark matter

In the formula for ρ_M^D, the term $\frac{\langle G\rho\rangle_D}{\langle G\rangle_D}$ accounts for baryons, while the term $-\frac{3}{8\pi}\frac{\langle F_2\rangle_D}{\langle G\rangle_D}$ mimics an effective dark matter component.

Physical picture

The additional fluids with ρ_Q^D and ρ_R^D can both provide a negative pressure and hence drive an accelerated expansion. The universe consists of baryons only, while inhomogeneities give rise to the full dark-side phenomenology. Comparison with data on background expansion and growth of structure is now in order. Hopefully, full G and Λ from the renormalization group.