Defeasible inclusions in low-complexity DLs: Preliminary notes

Piero Bonatti, Marco Faella, Luigi Sauro

Università di Napoli “Federico II”, Italy

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Overview

- 5-slides course in Description Logics (DLs)
- DLs and circumscription: general theory
- Circumscribing (middle/high)-complexity DLs: previous results
- Circumscribing low-complexity DLs: new results
- Conclusions and Future works
Description Logics

- A family of logics (closely related to Propositional Modal Logics) that enables to balance between expressiveness and complexity.
- A theoretical account for the Semantic Web (OWL).
DLs: interpretations

Alphabet:
- *concept names* (typical symbols $A, B$)
- *role names* (typical symbol $P$)
- *individual names* (typical symbols $a, b, c$).

Interpretations $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$:
- $\Delta^{\mathcal{I}}$ is a non-empty set of individuals (*domain*)
- $\cdot^{\mathcal{I}}$ (*interpretation function*) maps
  - $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
  - $P^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
  - $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$

An interpretation $\mathcal{I}$ is a *model* of a concept $C$ if $C^{\mathcal{I}} \neq \emptyset$. 
## DLs: syntax and semantics

### Usual constructors:

<table>
<thead>
<tr>
<th>Name</th>
<th>Syntax</th>
<th>Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>top</strong></td>
<td>( \top )</td>
<td>( \top^I = \Delta^I )</td>
</tr>
<tr>
<td><strong>bottom</strong></td>
<td>( \bot )</td>
<td>( \bot^I = \emptyset )</td>
</tr>
<tr>
<td>negation</td>
<td>( \neg C )</td>
<td>( \Delta^I \ \setminus \ C^I )</td>
</tr>
<tr>
<td>conjunction</td>
<td>( C \cap D )</td>
<td>( C^I \ \cap \ D^I )</td>
</tr>
<tr>
<td>disjunction</td>
<td>( C \cup D )</td>
<td>( C^I \ \cup \ D^I )</td>
</tr>
<tr>
<td>existential restriction</td>
<td>( \exists R.C )</td>
<td>( { d \in \Delta^I \mid \exists (d, e) \in R^I : e \in C^I } )</td>
</tr>
<tr>
<td>value restriction</td>
<td>( \forall R.C )</td>
<td>( { d \in \Delta^I \mid \forall (d, e) \in R^I : e \in C^I } )</td>
</tr>
<tr>
<td>nominal</td>
<td>{ a }</td>
<td>{ a^I }</td>
</tr>
<tr>
<td>inverse role</td>
<td>( R^- )</td>
<td>( (R^-)^I = { (d, e) \mid (e, d) \in R^I } )</td>
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</tbody>
</table>
Description Logics as Modal Logics

- **$\mathcal{ALC}$:**

  \[
  C, D ::= \text{A} | C \sqcup D | C \sqcap D | \neg C | \exists R.C | \forall R.C
  \]

- The reference logic $\mathcal{ALC}$ corresponds to normal (multi)modal logic

  \[
  \begin{align*}
  A & \iff a \\
  C \sqcap D & \iff \phi_C \land \phi_D \\
  C \sqcup D & \iff \phi_C \lor \phi_D \\
  \neg C & \iff \neg \phi_C \\
  \exists P.C & \iff \langle p \rangle \phi_C \\
  \forall P.C & \iff [p] \phi_C
  \end{align*}
  \]
A (strong) knowledge base $\mathcal{KB}$ is a finite set of

- **concept inclusions (CIs)** $C \sqsubseteq D$
- **concept assertions** $A(a)$ and **role assertions** $P(a, b)$
- **role inclusions (RIs)** $R \sqsubseteq R'$.

An interpretation $\mathcal{I}$ **satisfies**

- $C \sqsubseteq D$ iff $C^\mathcal{I} \subseteq D^\mathcal{I}$;
- $C(a)$ iff $a^\mathcal{I} \in C^\mathcal{I}$
- $R(a, b)$ iff $(a^\mathcal{I}, b^\mathcal{I}) \in R^\mathcal{I}$, and
- $R \sqsubseteq R'$ iff $R^\mathcal{I} \subseteq R'^\mathcal{I}$.

$\mathcal{I}$ is a **model** of $\mathcal{KB}$ iff $\mathcal{I}$ satisfies all the elements of $\mathcal{KB}$.

We write $C \sqsubseteq_{\mathcal{KB}} D$ iff for all models $\mathcal{I}$ of $\mathcal{KB}$, $\mathcal{I}$ satisfies $C \sqsubseteq D$. 
Nonmonotonic reasoning has several uses:

- As a communication convention. When a property is *typically* satisfied we are inclined to communicate only negative cases. This way communication is more efficient.
- As a rule of conjecture (typicality).
- As Database convention (Closed World Assumption).

Regarding Description Logics:

- Representing the *world* needs to abstract relevant properties and relations between entities.
- Abstraction process brings about a balance between generality/specificity of concepts, thus to maintain a sufficient degree of generality some properties are assumed to be *typically* satisfied in a class even if exceptions are tolerated.
The basic idea of circumscription is to introduce special atomic concepts that represent exceptional or abnormal states of affairs.

Abnormal concepts are intended to be minimized during reasoning as far as the knowledge base is satisfied.

This induces a preference relation over interpretations. Roughly, an interpretation $\mathcal{I}$ is preferred to an interpretation $\mathcal{J}$ if:

- both the interpretations satisfy the knowledge base
- the extensions in $\mathcal{I}$ of each abnormal concept is included in the corresponding extension in $\mathcal{J}$
- for at least one abnormal concept the inclusion is strict
An example

“Mammals normally inhabit land, whales do not live on land”

\[ \text{Mammal} \subseteq \exists \text{inhabit.Land} \cup \text{Ab}_{\text{Mammal}} \]
\[ \text{Whale} \subseteq \text{Mammal} \cap \neg \exists \text{inhabit.Land} \]
you could expect that the only those animals do not live on land for which it has been explicitly stated: whales.
this requires that the role inhabit can vary during minimization
Minimization with varying concepts

- Even more, you could expect that as the existence of abnormal mammals (i.e. whales) has not been explicitly stated they do not actually exist.
- This requires that also the concept *Whale* can vary during minimization.
In case a DL supports unqualified existential restrictions ($\exists R.\top$) variable roles can be used to represent also variable concepts. For each variable concept $A$ add a new role $P_A$ and replace any occurrence of $A$ in $KB$ with $\exists P_A.\top$. 
Conflicts between abnormal concepts

- Minimizing abnormal concepts may rise conflicts.

\[
\text{User} \sqsubseteq \neg \exists \text{hasAccessTo}.\text{ConfidentialFile} \sqcup \text{Ab}_{\text{User}} \\
\text{Staff} \sqsubseteq \text{User} \\\n\text{Staff} \sqsubseteq \exists \text{hasAccessTo}.\text{ConfidentialFile} \sqcup \text{Ab}_{\text{Staff}}
\]

- Abnormal concepts can be prioritized to resolve conflicts. This means to define a strict partial order \( \prec \) between abnormal concepts.
Circumscription settings can be specified by a *circumscription pattern* $CP = \langle \prec, M, F, V \rangle$ where:

- $M$ is the set of abnormal concepts
- $F$ is the set of fixed concepts
- $V$ is the set of variable concepts
- $\prec$ is a strict partial order over $M$

**Note**

- $M, F, V$ are a partition of $N_C$
- *We do not consider fixed roles to avoid undecidability*
Models of Circ<sub>CP</sub>(KB)

Definition (Models)

For all interpretations \( \mathcal{I} \) and \( \mathcal{J} \), and circumscription pattern \( CP \), let \( \mathcal{I} <_{CP} \mathcal{J} \) iff:

1. \( \Delta^\mathcal{I} = \Delta^\mathcal{J} \);
2. \( a^\mathcal{I} = a^\mathcal{J} \), for all \( a \in N_I \);
3. \( A^\mathcal{I} = A^\mathcal{J} \), for all \( A \in F \);
4. for all \( Ab \in M \), if \( Ab^\mathcal{I} \not\subseteq Ab^\mathcal{J} \) then there exists \( Ab' \in M \) such that \( Ab' \prec Ab \) and \( Ab'^\mathcal{I} \subset Ab^\mathcal{J} \);
5. there exists a \( Ab \in M \) such that \( Ab^\mathcal{I}(\delta) \subset Ab^\mathcal{J}(\delta) \) and for all \( Ab' \in M \) with \( Ab' \prec Ab \), \( Ab'^\mathcal{I} = Ab'^\mathcal{J} \).

Given a knowledge base \( KB \) and circumscription pattern \( CP = \langle \prec, M, F, V \rangle \), an interpretation \( \mathcal{I} \) is a model of \( \text{Circ}_{CP}(KB) \) iff \( \mathcal{I} \) is a (strong) model of \( KB \) and there exists no model \( \mathcal{J} \) of \( KB \) such that \( \mathcal{J} <_{CP} \mathcal{I} \).
Reasoning tasks

Let $\mathcal{KB}$ be a knowledge base and $CP$ a circumscription pattern. We consider the following standard reasoning tasks:

- Concept satisfiability: Given a concept $C$, check whether there exists a model $\mathcal{I}$ of $\text{Circ}_{CP}(\mathcal{KB})$ such that $C^\mathcal{I} \neq \emptyset$.

- Subsumption: Given two concepts $C$, $D$ check whether $\text{Circ}_{CP}(\mathcal{KB}) \models C \subseteq D$, that is, for all models $\mathcal{I}$ of $\text{Circ}_{CP}(\mathcal{KB})$, $C^\mathcal{I} \subseteq D^\mathcal{I}$.

- Instance checking: Given $a \in N_1$ and a concept $C$, check whether $\text{Circ}_{CP}(\mathcal{KB}) \models C(a)$, that is, for all models $\mathcal{I}$ of $\text{Circ}_{CP}(\mathcal{KB})$, $a^\mathcal{I} \in C^\mathcal{I}$.

Note

In ALC all these problems can be polynomially reduced to one another.
Previous results on circumscribed DLs

- Concept satisfiability is $\text{NExp}^{\text{NP}}$-complete for $\text{ALCQO}$ and $\text{ALCIO}$
- if the number of fixed and abnormal concepts is bounded to $n$, complexity of satisfiability drops to $\text{NP}^{\text{NExp}}$.

Note

$\text{NExp} \subseteq \text{NP}^{\text{NExp}} \subseteq \text{NExp}^{\text{NP}} \subseteq 2^{\text{Exp}}$

- Concept satisfiability with fixed roles is undecidable in $\text{ALC}$
- Concept satisfiability with minimizing roles is decidable in $\text{ALC}$ only with empty Tboxes.
Some considerations...

Enhancing DLs with circumscription is not cheap, even for middle-complexity DLs such as $ALCQO$. Strategies to manage complexity may include:

- Considering low-complexity DLs as underlying logics
- Allowing abnormal concepts to occur only in specific patterns
- Avoiding features which easily bring about undecidability (fixed, abnormal roles)
Currently low-complexity DLs are the most used in several contexts:

- RDF corresponds to DL-lite$_R$
- Biomedical ontologies (Galen, Snomed) can be almost entirely mapped in $\mathcal{EL}$-family.

In biomedical domain it is difficult to define reasonable conceptualizations without incurring exceptions. Example: “in humans, the heart is usually located on the left-hand side of the body; in humans with situs inversus, the heart is located on the right-hand side of the body”
Low-complexity DLs: syntax

- **$DL-lite_R$** restricts concept inclusions to expressions $C_L \sqsubseteq C_R$, where

  $$
  C_L ::= A \mid \exists R. \top \\
  C_R ::= C_L \mid \neg C_L
  $$

- **$\mathcal{E}L$:**

  $$
  C ::= A \mid \top \mid C_1 \sqcap C_2 \mid \exists P. C
  $$

- **$\mathcal{E}LHO$** extends $\mathcal{E}L$ with role hierarchy and nominals.
- **$\mathcal{E}L^{-A}$** extends $\mathcal{E}L$ with atomic negation.
- **$\mathcal{E}L\perp$** extends $\mathcal{E}L$ with $\perp$.
- **$LL \mathcal{E}L\perp$**

  $$
  A_1 \sqcap A_2 \sqsubseteq B \quad \exists P. T \sqsubseteq B \quad \exists P_1. T \sqsubseteq \exists P_2. B
  $$

  where $A$ and $B$ can be an atomic concept or $\perp$. 
Defeasible inclusions

- Abnormal concepts can occur in a knowledge base only implicitly by means of *defeasible inclusions* (DIs).
- A defeasible inclusion is an expression $A \sqsubseteq_n C$ whose intended meaning is: *A’s elements are normally in C.*

$$A \sqsubseteq_n C \iff A \sqsubseteq C \sqcup Ab_A$$

- A *defeasible knowledge base* (DKB) in a logic $\mathcal{DL}$ is a pair $(S, D)$ where:
  - $S$ is a strong $\mathcal{DL}$ knowledge base
  - $D$ is a set of DIs $A \sqsubseteq_n C$
The sentences: “in humans, the heart is usually located on the left-hand side of the body; in humans with situs inversus, the heart is located on the right-hand side of the body” can be formulated with the following $\mathcal{EL} \perp$ inclusions

\[
\text{Human} \sqsubseteq_n \exists \text{has_heart.} \exists \text{has_position.} \text{Left} \\
\text{Situs_Inversus} \sqsubseteq \exists \text{has_heart.} \exists \text{has_position.} \text{Right} \\
\exists \text{has_heart.} \exists \text{has_position.} \text{Left} \sqcap \\
\exists \text{has_heart.} \exists \text{has_position.} \text{Right} \sqsubseteq \perp
\]
Circumscribing DL-lite$_R$

- Given a model $\mathcal{I}$ of Circ$_p(\mathcal{KB})$ ($p = \text{var, F, fix}$) and $x \in \Delta_\mathcal{I}$, there exists a model $\mathcal{J}$ such that
  - $\Delta_\mathcal{J} \subseteq \Delta_\mathcal{I}$
  - $|\Delta_\mathcal{J}|$ is polynomial in the size of $\mathcal{KB}$
  - $x \in \Delta_\mathcal{J}$
  - for all concepts $C$, $x \in C^\mathcal{I}$ iff $x \in C^\mathcal{J}$
- Concept satisfiability is in $\Sigma^p_2$, whereas subsumption and instance checking are in $\Pi^p_2$
  - Guess a polynomial model $\mathcal{I}$ of $\mathcal{KB}$ that proves consistency or disproves subsumption/instance checking
  - With an NP oracle, check that $\mathcal{I}$ is minimal.
Any concept $C$ is satisfied by some model $\mathcal{I}$ of $\text{Circ}_x(\mathcal{ELHO})$ ($x = \text{var}, F, \text{fix}$)

defeasible inclusions cannot be blocked under $\text{Circ}_{\text{var}}(\mathcal{ELHO})$, circumscription collapses to classical reasoning.

- Instance checking and subsumption for $\mathcal{EL}$ are in $P$.

It is possible to reduce the Tbox satisfiability in $\mathcal{EL}^\neg A$ to the complement of subsumption in $\text{Circ}_F(\mathcal{EL})$

- subsumption and instance checking in $\text{Circ}_x(\mathcal{EL})$ ($x = F, \text{fix}$) are ExpTime-hard
Circumscribing the $\mathcal{EL}$-family (II)

- Tbox satisfiability in $\mathcal{EL}^{-A}$ can be reduced to the complement of subsumption in $\text{Circ}_{\text{var}}(\mathcal{EL}^\bot)$.
  - Subsumption and instance checking in $\text{Circ}_{\text{var}}(\mathcal{EL}^\bot)$ are ExpTime-hard.

- Similarly DL-lite$_R$, in LL $\mathcal{EL}^\bot$ we can restrict to models of polynomial size with respect to $\mathcal{KB}$.
  - Concept satisfiability are in $\Sigma_2^p$
  - Subsumption and instance checking are in $\Pi_2^p$
### Summary of complexity results

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<thead>
<tr>
<th></th>
<th>var</th>
<th>$F / \text{fix}$</th>
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<tbody>
<tr>
<td><strong>Concept sat.</strong></td>
<td>$\mathcal{EL}$</td>
<td>trivial up to $\mathcal{ELHO}$</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{EL}^\perp$</td>
<td>$\geq \text{ExpTime}$</td>
</tr>
<tr>
<td></td>
<td>$\text{DL-lite}_R, \ LL \mathcal{EL}^\perp$</td>
<td>$\leq \Sigma^p_2$</td>
</tr>
<tr>
<td><strong>Instance checking</strong></td>
<td>$\mathcal{EL}$</td>
<td>$P$</td>
</tr>
<tr>
<td><strong>Subsumption</strong></td>
<td>$\mathcal{EL}^\perp$</td>
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<td></td>
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Conclusions and Future Works

Conclusions

- Circumscribing low-complexity DLs is very sensible.
  - In some cases it reduces to classical reasoning ($\mathcal{ELHO}$)
  - In other case the complexity gap is impressive ($\mathcal{EL}^\perp$)
- However we don’t deal with strange beasts as $NP^{\text{NExpTime}}$

Future works

- Make results tighter
- Consider $\mathcal{FL}$-family
- Check whether fixed/abnormal roles lead to undecidability also in low-complexity DLs