Pointless Physics

Fedele Lizzi

Università di Napoli Federico II

and Institut de Ciencies del Cosmos, Universitat de Barcelona

Chicago 2019

In my elementary school book a **point** was defined as:

A Geometrical Entity without Dimension

I must confess that after reading it I was none the wiser about what a point is.

Probably because I was convinced I knew what a point is. I could produce them at will with my biro. Or better with a sharper pencil, or better with a pin, or better. . .

Maybe things are not so simple. . .

Euclid defined a point as **That which has no part**

The highest authority I can think of, *Wikipedia*, states:

In Euclidean geometry, a point is a primitive notion upon which the geometry is built.

Which fits well with what I was thinking in first grade, but it is not satisfactory at all ages. . .

Let's get to the point.

Points are ubiquitous in both physics and mathematics, but are we sure we are always talking of the same thing?

We usually think of points as something elementary and therefore "small". In physics this may be misleading.

In astrophysics a point may be a galaxy, or even a cluster of galaxies.

Before relativity points where sometime concerned purely with space, an absolute entity. "Things" were occupying a set of these points, as an idealisation a single point.

Dynamics a change of these positions in a continuous and smooth way.

With special relativity we have a first drastic change. Points become **events.** Time and space are now fused into a single entity:spacetime.

The second change comes with general relativity. Gravity **curves** spacetime.

There are two aspects I want to stress here:

- General relativity is not just a theory which "uses" geometry. In Einstein's vision gravity **is** geometry

- The points of these geometry are all possible events, this implies that there is some structure relative to them, a clock, a rod, something (at least potentially) operational

As you know in parallel to the development of general relativity there was the development of **Quantum Mechanics**.

Let me backtrack a bit and discuss classical dynamics. Here we encounter another set of “points

Newton realised that in order to describe the motion of point particles he needed to know the initial position, and the initial velocity, as well as the set of forces acting on the particle.

Let me open here a parenthesis, he delayed for year the publication of the Principia because he had to convince himself that he could treat a planet, big as it is, as a single point. Thus introducing a first novel interpretation of points.

Newton introduced what we now call the phase space, i.e. the identification of a geometry in which points are identified with the state of motion of a particle (thanks to Decartes we could give names to them).

I am not sure, but this could have been the introduction of higher dimensional geometries.

Newtonian Mechanics is based on the knowledge of the state of motion, a point in phase space, a set of forces and an absolute time, which acts as parameter for the trajectory of these points.

To describe motion Newton had to introduce differential calculus, thus giving structure to Euclidean and Cartesian spaces.

It would be interesting to connect the points in the Descartes and Newton geometry with the Monads of Leibniz, and earlier of Giordano Bruno. Lack of time and competence prevent me from doing this.

The likes of Lagrange, Laplace, Hamilton and many more understood various aspects of dynamics developing what we now call classical mechanics, which can be seen as the geometry of phase space.

This can be very intricate in system with constraints, or have very high dimensionality. A state of a mole of gas is described by a point in a space with $\boxed{12 \cdot 10^{23}}$ dimensions.

It was quickly realised that points in the phase space of a gas have not operational meaning, and that they were not really the point, and that a gas could be well described in a much smaller space of a few variables, like volume, temperature, pressure.

Still in classical mechanics this is just an effective description, a single mole of gas at a given instant is described by a single point in a huge phase space. Just that there are several points so similar to it, that knowledge of the precise point is uninteresting, other than impossible to gather.

Also for systems with few (even one) particle it may impossible to precisely know the position and velocity of a particle. Whichever measurement we make will be marred by experimental errors. At most, rather than a precise position and velocity, we may measure a probability density.

This is a function usually with a “bell shape” (a Gaussian). Nothing prevent us from conceiving a set of more and more precise instruments, which could give us more and more precise measurement.

Another issue is the fact, known already to Poincaré at least, that very close initial points may, evolved in time, give rise to vastly different trajectories, giving rise to chaotic phenomena.

The Gaussian may become sharper and sharper and in the limit localise precisely at a point. Physicist call this “function” a Dirac $\boxed{\delta}$.

Whatever the theoretical, technological or computational problems, in classical mechanics points are there, as a real entity.

And you know that at the beginning of last century things changed dramatically with the appearance of two fundamental quantitites...

$$\boxed{c}$$

$$\boxed{\hbar}$$

Planck's constant and the speed of light are two dimensionful constant of nature, unlike for example $\boxed{\pi}$ or Napier's $\boxed{e}$.

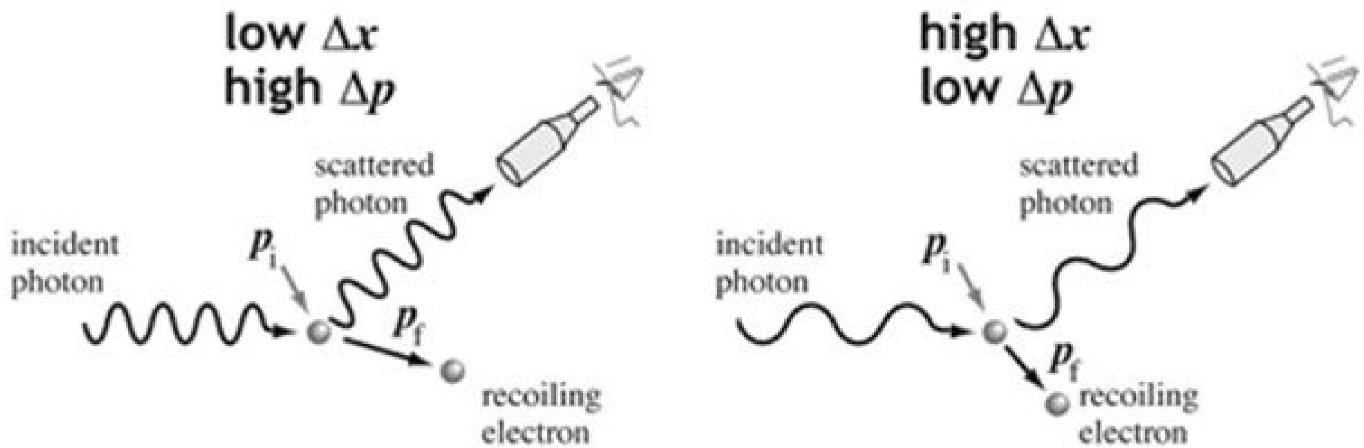
In particular $\boxed{\hbar}$ has the dimension of an area in phase space (length $\boxed{\times}$ velocity $\boxed{\times}$ mass).

In particular it may define some sort of minimal area in phase space. Not in the sense of a discrete space like a lattice, but in a subtle way, the impossibility to localise a particle in phase space because of the Heisenberg principle.

I will justify it with an argument called Heisenberg's Microscope. It uses the fact that, light is made of quanta, the photons, which have a finite size (wavelength) which is inversely proportional to their energy.

$$\Delta x \, \Delta p \geq \hbar$$

Δx = uncertainty in position
Δp = uncertainty in momentum
$\hbar$ = $h / 2\pi$



The idea is that to "see" something small, of size of the order of $\boxed{\Delta x}$, we have to send a "small" photon, that is a photon with a small wavelength $\boxed{\lambda}$, but a small wavelength means a large momentum $\boxed{p = h/\lambda}$. In the collision there will a transfer of momentum, so that we can capture the photon. The amount of momentum transferred is uncertain.

If one does the calculation using the resolving power of an ideal microscope one finds:

$$\boxed{\Delta x \Delta p \geq h}$$

where $\boxed{h}$ is Planck's constant.

The argument is very heuristic, and the result is off by an order of magnitude ($\boxed{4\pi}$). We know that in order to obtain the uncertainty principle it is necessary to have a solid theory, quantum mechanics, where $\boxed{p}$ and $\boxed{q}$ become operators, and then it is possible to prove:

$$\boxed{\Delta x \Delta p \geq \frac{\hbar}{2}}$$

I dare not affirm that quantum mechanics is well understood, but I venture to say that we have a solid mathematical framework with which we can work.

Most of the mathematics necessary was developed by Weyl, Wigner, Von Neumann (who also coined the expression "pointelss mathematics"), and from a pure mathematical point of view Gelfand, Naimark, Moser, Connes...

Without being technical the idea is that for ordinary geometry (and topology) all of the information relative to points, and their relations, can be encoded in the fields, or functions defined on the space.

These functions are usually commutative, namely $\boxed{f \cdot g = g \cdot f}$, and with the help of other mathematical structures (derivatives for example) it is possible to describe things like distances, tangents, areas. . .

In the formalism of quantum mechanics position $\boxed{x}$ and momentum $\boxed{p}$, which are themselves functions, do not commute:

$$\boxed{x \cdot p - p \cdot x = i\hbar}$$

It is possible to prove that Heisenberg's principle is a consequence of this noncommutativity

From this the name of **Noncommutative Geometry**

All this was for phase space, but position space is not affected by noncommutativity of uncertainty. Taking two coordinates

$$\boxed{x \cdot y = y \cdot x \text{ and } \Delta x \Delta y \text{ can be zero}}$$

If we are willing to give up information about velocity, it is possible to localise points of space in quantum mechanics

And things do not change for relativistic theories and spacetime localisation.

Let me introduce the third fundamental quantity:

$$\boxed{G}$$

The gravitational constant gives the strength of the force of gravity, and how much spacetime curves in the presence of mass

What concerns us is that we can combine the three dimensionful quantities to obtain a fundamental Length, called Planck's length:

$$\boxed{\ell = \sqrt{\frac{\hbar G}{c^3}} \simeq 10^{-33}\text{cm}}$$

There is a phenomenon noticed probably for the first time by Bronstein in 1938, but presented independently in a modern and most terse way by Doplicher, Fredenhagen and Roberts in 1994.

Let us repeat the Heisenberg microscope thought experiment without caring about the subsequent state of motion of the particle we want to localise.

We take more and more energetic probes, and concentrate energy is smaller and smaller volumes to see if something is there.

At some point however we have put too much energy (which in relativity is the same as mass) in too small a volume. At this point a (microscopic) black hole is formed, and anything inside the horizon cannot be measured anymore.

For a rigorous statement we would need a full theory of **quantum gravity**. A theory which do not have. At least not yet. . .

While we do not have a full fledged theory of quantum gravity, several people will nevertheless make statements like ”*the most serious candidate for such a theory is . . .*” Different people will fill the dots in different ways.

The authoritative Wikipedia lists 25 of these approaches, mostly chunked in a section called “other approaches”. Only two of them deserve a full fledged section: Strings and Loop Quantum Gravity.

Among the various approaches it appears also Noncommuative Geometry, although I do not have very clear what is meant.

Remember that in general relativity spacetime becomes not only curve, it becomes dynamical. This creates a clash with the most advances form of quantum mechanics: Quantum Field Theory.

Quantum fields are functions which represent the various particles after quantization. These fields, and their interaction, have a peculiarity, their values depend on the scale of the energy, and therefore of the distances.

In doing calculations often one encounter some infinities, in the form of divergent quantities, like

$$\boxed{\lim_{r \to 0} \frac{1}{r}}$$

which would lead to a division by zero. These are called ultraviolet (highe energy/small distance) divergences.

These infinities are however under control with renormalization. Whereby one learns how to subtract these infinities to get finite quantities.

In the usual quantum field theory these ultraviolet divergences do not pose problems for the concept of point. Fields are still defined on a regular (flat) spacetime.

Divergences and renormalization may still need some mathematical polishing, but the success of the theory is fantastic.

Quantum electrodynamics makes predictions which are verified with **14** digits accuracy, and *CERN* has found basically all predicted particles.

Still, the scales at the Large hadron Collider are of the order of $\boxed{10^{-20}\ \mathrm{m}}$. This is still $\boxed{15}$ orders of magnitude away from Planck Lenght $\boxed{10^{-35}\ \mathrm{m}}$. We are far from quantum gravity.

Converting in energies, LHC is at an energy of $\boxed{10^{4}\mathrm{GeV}}$, compared with Planck's Energy of $\boxed{10^{19}\mathrm{GeV}}$

The problem is that the renormalization programme does not work of the field one is quantising is spacetime itself.

This statement requires some clarification, the field that one can quantize is the the metric, which gives curvature and distance among points. Like every field there is a particle associated to this field, called the graviton.

The graviton, when interacting with other particles and itself, behaves in a crazy way, so that the usual subtraction of infinities does not male sense.

We do not see this because the scale at which we can presently do experiments are too small to notice these effects, and the theory we have serves us well

It is the same as quantum mechanics or relativity with respect to classical mechanics. A long as we are doing experiment at scales far from $\boxed{c}$ or $\boxed{\hbar}$, classical mechanics is a perfectly viable theory.

All this points to the need for some sort of

$$\boxed{\textbf{Quantum Spacetime}}$$

Which mathematical structure will describe it?

Attempts are under way.

A possible spacetime is one which comes out of a replica of what has been done for phase space, take coordinates of spacetime and make them non-commutative

Such an approach emerged in some limit of string theory and, mostly due to a paper of Seiberg and Witten, was very fashionable for some time. To the extent that to this date for some noncommutative geometry is a synonym of the above relation.

It is not. If anything strings could give an example of quantum space.

In this approach fields are still functions of some sort of spacetime, but the product among functions is a deformed, noncommutative $\boxed{\star}$-product.

$$\boxed{x \star y - y \star x = \theta}$$

with $\boxed{\theta}$ a constant.

This begs the question: are points there or not? After all to define this deformed product we have to define functions on points.

This is a discussion I have been having with Nick Huggett for some time.

In my opinion points are not there, because, just like for phase space in quantum mechanics, for which the $\boxed{\star}$-product was introduced, it is impossible to operationally measure points below the threshold given by Planck scale

The physics tells us that there are strange phenomena if one closes up on point beyond this threshold, for example a curious mixing between short and long scales.

Mathematically, if I consider functions which are too sharply peaked, and multiply them, the resulting function oscillates wildly.

Points might still be there. But are there for us to make sense?

After all, if you may need a ghost in the machine, you also need a machine in the ghost to "know" that something is there. And there may be no machine which may find them.

In general in string theory spacetime is quantized. Coordinates are fields on a two dimensional surface (the world sheet) traced by the string moving in spacetime.

In loop quantum gravity spacetime becomes a foam of spin relations. The proposal is radical but, oversimplifying, it suffer from the problem that it is difficult to see how, and in which limit, ordinary spacetime emerges.

I will not discuss other approaches, and finish with a more strict noncommutative geometry view of things, and describe a model in which point again loose their meaning.

Taking the cue from quantum mechanics, and Hilbert's programme, I like to see geometry, quantum or otherwise, in a way inspired by Heisenberg's matrix mechanics, or the theory of operators on Hilbert spaces.

This Alain Connes programme, for which everything in in the spectral properties of the matrice.

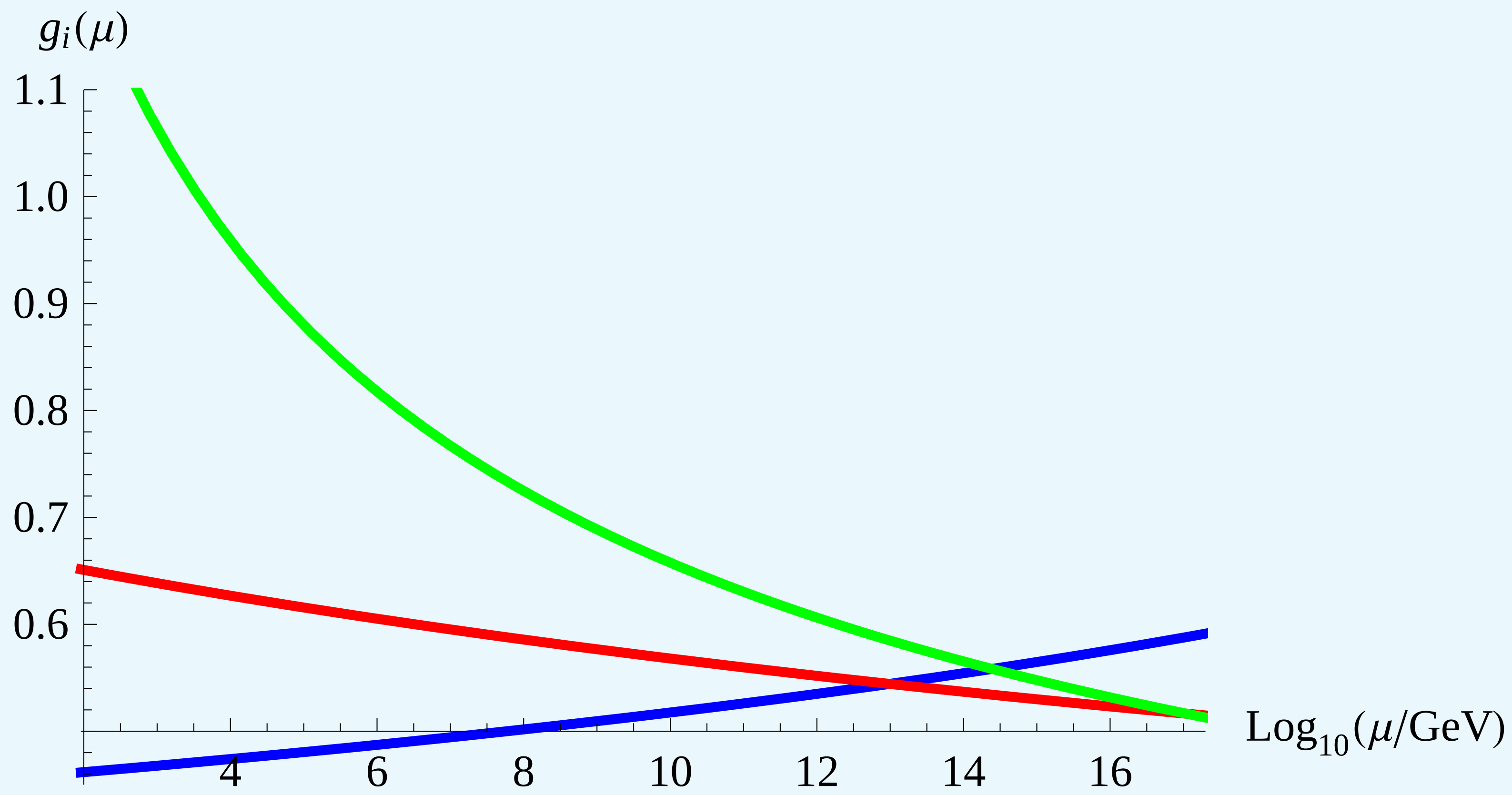
The spectrum, which is a set of numbers, captures the charateristic of the matrices, independently from their transformations, like for example rotations.

Ordinary geometry can be described by the spectra of these operators in the case they form a commutative algebra. But the formalism is ready to describe noncommutative cases, such as quantum mechanics

With these techniques one can describe field theory as well, writing everything in terms of these operators.

Presently it is impossible to even conceive an accelerator experiment which may probe Plank's energy. Some hope may come form observation of cosmic rays, of the cosmic background, and in the future from gravitational waves.

In the meantime we may however "see things from below". As I said, because of renormalization some quantities depend on the scale, and among them the strength of the fundamental interactions (excluding gravity:)



$g_i(\mu)$
1.1
1.0
0.9
0.8
0.7
0.6
4
6
8
10
12
14
16
$\mathrm{Log}_{10}(\mu/\mathrm{GeV})$

This picture is valid in the absence of new physics, i.e. new particles and new interactions which would alter the equations which govern the running

The three interaction strength start from rather different values but come together almost at a single unification point

But then the nonabelian interactions proceed towards asymptotic freedom, while the abelian one climbs towards a Landau pole at incredibly high energies $\boxed{10^{53}\ \text{GeV}}$

The lack of a unification point was one of the reasons for the falling out of fashion of GUT's.

Some supersymmetric theories have unification point

We all believe that this running will be stopped by “something” at $\boxed{10^{19}\ \text{GeV}}$

This unknown something is quantum gravity

I take the point of view that there is a fundamental change of the degrees of freedom of spacetime. And will try to use Noncomutative Spectral Geometry to infer something

In particular, together with Kurkov and Vassilievich, we performed an expansion not around the small energy as is usually done, but using instead infinite energy as the point around which we deform the theory.

Technically the elimination of infinities is performed introducing a cutoff, i.e. choosing an energy which forces the matrices to be finite.

Usually point “talk to each other” with an exchange of fields. They are this way correlated. For example if the field is electromagnetism, in three plus one dimensions, the force among two charges decreases as the inverse of the distance, if the energy is low.

And this is way to probe geometry

We took instead the opposite point of view, and found that, in the limit (unattainable of course), point become completely uncorrelated from each other.

It is like the phenomenon (present in an uncorrelated research) of asymptotic silence, for which the causal light comes of particles degenerates into single lines, and no communication is possible.

This is a limiting behaviour, I think one has to take it as a general indication that the presence of a physical cutoff scale in momenta leads to a "non geometric phase" in which the concept of point ceases to have meaning, possibly described by a noncommutative geometry

Note that throughout this discussion I have done nothing to spacetime, I have only imposed the cutoff and used standard techniques and interpretations

Points may still be there, but they are uncorrelated at very high energy.

Clearly this indicates a phase transition for which the locality order parameter (position) is not present anymore.

But it also gives us the indication that quantum gravity must be a theory in which points are not a relevant entity.

This is coherent with all other indications.

It is like a deep water fish trying to understand what goes on above his ceiling.

He knows that pressure decreases as he goes up. He can also infer some properties of a different states of matter by looking at bubbles which are creates near some “high energy” volcanic vents or when “above” there are storms, but he cannot naturally grasp the concept of air, or absence of water.

He will need a higher leap