Time to think about time?

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The aim of this half-talk is to stimulate some discussion of the issue of the Lorentz vs. Euclidean settings, mostly in the Connes approach to the standard model.

I will be very sketchy and try to give just the flavour. Some references at the end may help fill the gaps.

The main part will be devoted to the connections between twisted spectral triples and Lorentzian signature field theory.

The description of the standard model of particle interactions based on NCG is based on the following spectral triple

- The algebra is the product of functions on spacetime times a finite algebra whose unimodular elements give the gauge group: $\boxed{\mathcal{A} = C(M) \otimes \mathcal{A}_F}$ with $\boxed{\mathcal{A}_F = C(M) \otimes (\mathbb{C} \oplus \mathbb{H} \oplus \mathsf{Mat}_3(\mathbb{C}))}$

- The Hilbert space is the product of spacetime spinors times an internal space describing all 96 fermionic matter degrees of freedom of quarks and leptons: $\boxed{\mathcal{H} = \mathsf{Sp}(M) \otimes H_F}$

- The Dirac operator is the sum of the usual one (with the Levi-Civita connection) and a term acting on the fermions, the latter brings the informations on the masses and mixing: $\boxed{D_0 = (\partial\!\!\!/ + \omega\!\!\!/) \otimes \mathbb{I} + \gamma_5 \otimes D_F}$

- The Even (Chirality) and Real (Charge Conjugation) structures are the product of the usual ones times an internal component: $\boxed{\Gamma = \gamma_5 \otimes \gamma_f}$ and $\boxed{J = j \otimes J_F}$

The coupling with a background is done adding to D_0 a potential, i.e. a connection one-form, defined as $D = D_0 + \not{A} = D_0 + \sum_i a_i[D_0, b_i]$, with $a_i, b_i \in \mathcal{A}$
This gives an extra field, the Higgs.

The action is the sum of the bosonic spectral action plus the fermionic action:

$$S = \operatorname{Tr} \chi\left(\frac{D_A^2}{\Lambda^2}\right) + \langle J\Psi | D\Psi \rangle$$

where Λ an energy cutoff scale, and χ is a cutoff function.

With these data it is possible to calculate the mass of the Higgs.

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The number is wrong.

It is therefore necessary to improve the model. This is possible, at the price of loosing predictivity.

From the physical point of view there are a few shortcomings of the model, understanding these, and making the model more realistic will improve things. Let me quote a few of them.

In particular spacetime has to be compact and Euclidean. I will concentrate on this second aspect. Nothing, as far as I know, has been done for the fist, although this also may lead to interesting developments, especially in view of the fact that the "infrared frontier" is gaining attention.

Physicists are of course also guilty of the same. To regularize infrared divergences systems are "put in a box", and by Wick rotating to Euclidean space the convergence properties of functional integrals is improved, thus allowing to perform calculations. At the end the size of the box is sent to infinity, and the system is rotated back to Lorentz signature.

Often Wick rotation is presented as an analytic continuation of time to the imaginary axis $t \to \mathrm{i}t$. But there is much more to it, some of which is not totally understood.

On a curved background more correctly one has to rotate vierbeins $e^0_\mu \to \mathrm{i} e^0_\mu$

This is fine for bosons and the bosonic action. But for fermions and the fermionic actions things are very different.

The group $\mathrm{Spin}(1,3)$ is different from $\mathrm{Spin}(4)$, γ matrices, generators, charge conjugation, are different. Also the fermionic action changes, since the quadratic forms have to be invariant under the proper group transformations

$$\bar{\psi}\, \gamma^A_{\mathsf{M}}\, e^\mu_A \left(\left[\nabla^{\mathsf{LC}}_\mu \right]^{\mathsf{M}} + \mathrm{i} A_\mu \right) \psi, \qquad \bar{\psi}\psi$$

$\bar{\psi} \equiv \psi^\dagger \gamma^0$ and ∇^{LC}_μ the covariant derivative on the spinor bundle with the Levi-Civita spin-connection, which is different for Lorentzian and Euclidean

The corresponding terms with the required Spin(4) invariance are:

$$\psi^\dagger\, \gamma^A_{\mathsf{E}}\, e^\mu_A \left[\nabla^{\mathsf{LC}}_\mu \right]^{\mathsf{E}} \psi, \qquad \psi^\dagger \psi$$

And another important aspect is the fact that in the Euclidean case the degrees of freedom are doubled, since $\boxed{\psi, \psi^\dagger}$ are independent. This is a realtive of the Osterwalder-Schrader doubling in second quantization.

This is actually very nice in the context of the NCG standard model, where a doubling of the degrees of freedom is unavoidably present.

I have no time to go into details, but it turns out that when Wick (anti) rotating to the Lorentz signature, this spurious doubling is naturally eliminated. Making the fermion doubling, which was perceived as a problem, a natural features of the model.

Still one has to notice that usually the starting point is a theory with Lorentzian signature, which is rotated, and after calculations one goes back. In our case the theory is defined in the Euclidean, hence there is little guidance as to "go back".

This led to the study of Krein space rather than Hilbert spaces. In this case the inner product $\langle\cdot|\cdot\rangle$ is not positive definite, but there is an operator R such that $\langle\cdot|R\cdot\rangle$ is positive definite.

In the following I wish to connect Lorentz signature and Krein spaces to a known variant of noncommutative geometry: Twisted Spectral triples.

These we introduced by Connes and Moscovici to incorporate type III algebras coming from foliations. The idea in this context is to replace the usual commutator with a twisted commutator, for example for the connections

$$[D, a]_\rho := Da - \rho(a)D$$

Where ρ is an algebra automorphism

One can then repeat the construction (with important steps which I skip) leading to the action and so on.

The interest of twisting the spectral triple of the standard model has to do with the improvement necessary to obtain the correct mass of the Higgs

An extra field is needed in the position occupied by the right handed neutrino, but this field would not emerge in the usual construction. It does however emerge from an (enlarged) twisted triple.

Let me introduce a twisted inner product with the property

$\langle \Psi, \mathcal{O}\Phi \rangle_\rho = \langle \rho(\mathcal{O})^\dagger \Psi, \Phi \rangle_\rho$ where $\rho(\mathcal{O})^\dagger \equiv \mathcal{O}^+$ is the adjoint of $\rho(\mathcal{O})$ with respect to the initial Hilbert inner product.

Consider ρ to be inner, i.e. $\rho(\mathcal{O}) = R\mathcal{O}R^\dagger$, with R unitary.

Further conditions must be imposed on $RJR^\dagger$ and unbounded operator must be handled with care.

Before I go ahead let me remind a few facts. The Hilbert space of particles is split into the two eigenspaces of Γ. The real structure J flips the two (and switches to the opposite algebra.

It turns out that the inner automorphism necessary for the standard model is exactly conjugation by one of Euclidean Dirac γ matrices, which we usually call $\gamma_0^E = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & \mathbb{0} \end{pmatrix}$. Notice also that γ_0 is the only γ which is the same also in the Lorentz signature.

Everything then follows as simple exercises. The twisted inner product is the usual product introduced by Dirac with $\bar{\psi} = \psi^\dagger \gamma_0$, which is a Krein product with $R = \gamma_0$.

The fermionic action becomes the Lorentzian signature action in a nontrivial way, but some care is necessary for the spectral action

If one then writes the bosonic action as $\mathrm{Tr}\,\chi\left(\frac{\rho(D_A)D_A}{\Lambda^2}\right)$, nearly miracolously one obtains (in the free case) the standard model action (in flat space) with the γ matrices corresponding to the Lorentz signature!

Conclusions

Twisted spectral triple haver an intrinsic mathematical interest. But their naive application to a particular physical model have yelded several unexpected bonuses.

The setting used was still Euclidean, I never introduced a metric with a Lorentz signature, and the Hilbert space with the positive definite inner product is still there. We just noticed that all of the formulae of the Euclidean context were becoming the Lorentzian ones after the necessary twist.

This may be a curiosity, or it may point to a different approach to the introduction of a Lorentzian signature based on a transformation of the spectral triple, so that the Krein structure emerges as a derived concept.

Even more ambitiously, if we are able to built a noncommutative geometry of a second quantized theory, the concept of time may emerge from a twisted structure.

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