Inconstants

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This talk has also the duty to introduce the round table which will follow on "NC Geometry and deformation quantization approaches

From physics point of view deformation quantization started to understand quantum mechanics, the space being deformed is the pause space. While it is still largely used to this extent, later, following the work of Doplicher, Fredenhagen and Roberts, the object to deform has become spacetime

This second point of view has proven very fruitful for physics, especially after the work of Seiberg and Witten, and there has been a flurry of activity in this direction. This has produced good physics, and also some good mathematics.

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In the course of the round table I will briefly discuss the recent uses of deformation quantization (in a very ample sense) for spacetime, but in this talk I will specifically present some work I did with my collaborators on quantization of phase space and space-time. Unlike he previous talk on strict quantization, I will instead be in the just introduced framework of lenient quantization.

Physics is largely about *scales*, namely (usually) dimensionful quantities which determine how strong is an interaction, how fast is some radiation, how energetic is an atom. Those constants appear in the fundamental equations which describe the world

Three of those are considerate fundamental, and they are Planck's quantum of action $\boxed{\hbar}$, the speed of light $\boxed{c}$, and the Gravitational constant $\boxed{G}$

Combining them we can build scales which "measure" things:

- Length $\boxed{\ell_p = \sqrt{\frac{\hbar G}{c^3}} \simeq 10^{-35}\text{m}}$

- Time $\boxed{t_p = \sqrt{\frac{\hbar G}{c^5}} \simeq 10^{-44}\text{sec}}$

- Mass $\boxed{m_p = \sqrt{\frac{\hbar c}{G}} \simeq 10^{-8}\text{Kg}}$

In a Planckian regime of energies $\boxed{E_p = \sqrt{\frac{\hbar c^5}{G}} \simeq 10^{19}\text{GeV}}$ both quantum and general relativity effects are relevant.

One should use a full fledged theory of *quantum gravity*, a theory which sadly we do not yet have.

We can nevertheless surmise that at high energies/short distances there is a rich structure, and some theories point towards some sort of "granularity".

In the following I will first investigate the possibility that Planck's constant $\boxed{\hbar}$ is a quantity rapidly stochastically varying. Work in collaboration with Mangano and Porzio

I will then consider a similar scenario for Newton's constant $\boxed{G_{\mathrm{N}}}$. Work in collaboration with De Cesare and Sakellariadou.

I will be sketchy and skip the details of the calculations, which are present in the papers.

A comment:

We not the first to consider the fact that fundamental constants are varying quanitites

The idea goes back to Dirac and his *Large Number Hypothesis* in 1937.

It has been further developed in the following year, and there are some stringent limits on their variation.

The variations hypothesised by Dirac, and experimentally tested, are however on a very large (cosmological) time scale.

To my knowledge we are the first to consider these variations to be stochastic with a very short (Planckian) scale.

In the case of $\boxed{\hbar}$ our starting point can be a general noncommutative spacetime describe by the commutation relations:

$$\boxed{[x^i, p_j] = iH^i_j} \qquad \boxed{[x^i, x^j] = i\theta^{ij}} \qquad \boxed{[p_i, p_j] = iC_{ij}}$$

With $\boxed{H, \theta}$ and $\boxed{C}$ generic, i.e. non constant. Usually however $\boxed{H^i_j = \hbar\delta^i_j}$ is chosen to be constant.

The last two relations break Lorentz invariance, unless they are themselves randomly varying, oscillating around zero. In this caase Lorentz invariance is recovered in an effective way as an average.

It is most natural to immagine che the correlation lenght and time of a variable concerning quatum space time be of Planckian nature

How is space(time) measured?

I will discuss the question in the context of non relativistic quantum mechanics. A treatment using quantum field theory and/or relativity is more ambitious, but not unreachable

Starting points are the observables. The selfadjoint part of an algebra of operators on an Hilbert space.

A state of a physical system is a map from the algebra which is positive and of unit norm. Pure states (which cannot be written as convex sum of other states) are the vectios of the Hilbert space, the rest of the states are represented by mixed density matrices

Usually as algebra we take the (bounded) operators functions of $\boxed{\widehat{x}}$ and $\boxed{\widehat{p}}$

In this view configuration space emerges as the selfadjoint part of a commutative subalgebra, in other word the algebra generated by $\boxed{\widehat{x}}$ alone. From this commutative algebra it is possible to reconstruct the topology of configuration space as the set of pure states of this commutative algebra

Considering all of the variables of space time as a single vector, $Y^A = \{x^i, p_j\}$, $A = 1 \dots 2d$ the commutations relations can be stored into a single antysymmetric $2d \times 2d$ matrix

$$\Omega = \begin{pmatrix} \theta^{ij} & H^i_j \\ -H^i_j & C_{ij} \end{pmatrix}$$

It is always possible, at least locally, to put the matrix Ω in canonical form with a Darboux transformation to obtain:

$$\Omega' = \begin{pmatrix} 0 & {H'}^i_j \\ -{H'}^i_j & 0 \end{pmatrix}$$

This suggest to consider the modified commutation relation

$$[x^i, p_j] = {H'}^i_j$$

Which is tantamount to having a *Planck' Inconstant*

In the following we will concentrate on a simple one dimensional model

$$[\mathbf{x}, \mathbf{p}] = i\hbar(1 + \varepsilon(t))$$

with ε rapidly changing with time

$$\overline{\varepsilon(t)} = 0 \quad ; \quad \overline{\varepsilon(t)\varepsilon(t')} = \tau\, \delta(t - t')$$

Overline denotes the mean over ε probability distribution. The fluctuations are uncorrelated for time differences larger than a typical correlation time τ

The time evolution of an operator is $\boxed{\frac{d\mathbf{A}}{dt} = \frac{1}{i\hbar}[\mathbf{A},\mathbf{H}] + \frac{\partial \mathbf{A}}{\partial t}}$

The dependence on $\boxed{\varepsilon}$ is given by the commutator. The Poisson bracket, whose quantization gives the commutator, is also fluctuating.

This is coherent with the view that the fluctuations are an effective way to take into account an underlying structure

We now need to represent $\boxed{x}$ and $\boxed{p}$ as operators reflecting the modified commutator

$$\boxed{\mathbf{x}\,\psi(x) = A(t)\,x\,\psi(x) = A(t)\,\mathbf{x}_0\,\psi(x)}$$

$$\boxed{\mathbf{p}\,\psi(x) = -i\hbar\,B(t)\,\frac{d}{dx}\,\psi(x) = B(t)\,\mathbf{p}_0\,\psi(x)}$$

with $\boxed{A(t)B(t) = 1 + \varepsilon(t)}$, and $\boxed{\mathbf{x}_0,\ \mathbf{p}_0}$ the canonical pair of standard quantum mechanics.

We treat position and momentum on a par $\boxed{A(t) = B(t) = \sqrt{1+\varepsilon(t)}\,,}$

Such an effective variable $\boxed{\hbar}$ will undoubtedly have consequences at several levels. The effects will depend on the scale $\boxed{\tau}$

In our paper we investigated two possible experimental signatures. Surely there will be many more, and we hope that other groups will explore other possibilities. We looked at

- Free particles and interferometric experiments.

- Harmonic Oscillators and coherent light

The Schrödinger equation for the free particle will show a time dependence via $\varepsilon(t)$

$$i\hbar\frac{\partial}{\partial t}\psi = \frac{1}{2m}(1+\varepsilon(t))\,\mathbf{p}_0^2\,\psi$$

It is possible to solve it for a plane wave of momentum p_0

$$\psi_{p_0}(x,t) = \frac{1}{\sqrt{2\pi}}\exp\left[i\frac{p_0 x}{\hbar} - i\frac{p_0^2}{2m\hbar}\left(t + \int_0^t \varepsilon(t')dt'\right)\right]$$

When measuring an observable in this scheme there are two averaging processes, conceptually distinct.

- Averaging over the time fluctuations of $\boxed{\varepsilon}$: $\boxed{\bar{\mathbf{A}}}$

- The quantum mechanical averaging.The possible results of a measurement are the eigenvalues of the operator with a probability given by the state: $\boxed{\langle \mathbf{A} \rangle}$

In practice, for $\boxed{\tau}$ smaller than the experimental time resolution the two averaging coincide. Repeating the experiment samples both distribution. Nevertheless they are conceptually different, and I will keep the two notations

For a gaussian peaked at $\overline{p}$ and variance δ^2 we have

$$\psi(x,t) = \int \frac{dp_0}{\sqrt{2\pi}} \frac{1}{(\pi\delta^2)^{1/4}} e^{-\frac{(p_0-\overline{p})^2}{2\delta^2} + ip_0 x/\hbar - ip_0^2\left(t + \int_0^t \varepsilon(t)\right)/(2m\hbar)}$$

The mean distance travelled by a particle is the usual $\overline{\langle \mathbf{x} \rangle}_\psi(t) - \overline{\langle \mathbf{x} \rangle}_\psi(0) = \frac{\overline{p}}{m} t\}$

While the uncertainty is

$$(\Delta \mathbf{x})^2_\psi(t) - (\Delta \mathbf{x})^2_\psi(0) = \frac{\delta^2}{2m^2} t^2 + \frac{\overline{p}^2 + \delta^2/2}{m^2} \tau\, t$$

The motion is like a Brownian motion with diffusion coefficient

$$D = \frac{\overline{p}^2 + \delta^2/2}{2m^2}\tau$$

For $\delta \ll \overline{p}$, one can view D as due to *scatterings* with mean free path $(\overline{p}/m)\tau$.

Scattering over the quantum structure of spacetime

The usual spreading of the wave packet will dominate, but the effect can be enhanced for massless particles. In this case

$$(\Delta \mathrm{x})^2_\psi(t) - (\Delta \mathrm{x})^2_\psi(0) = c^2\,\tau\,t$$

It is possible to measure this effect in a double slit experiment

Waves are detected at some fixed distance L from the plate, the effect is a change δt of travel time with variance

$$\overline{\delta t^2} = \tau\, t = \tau\, L/c$$

with $t = L/c$ the time mean value

For frequency ω and intensity I at the mid-point on the screen

$$I \propto \frac{1}{4} \left| e^{-i\omega(t+\delta t_1)} + e^{-i\omega(t+\delta t_2)} \right|^2 = \frac{1}{2} \left(1 + \cos\left[\omega(\delta t_1 - \delta t_2)\right]\right)$$

$\delta t_{1,2}$ are the uncorrelated time shift along the two paths.In the standard case the two waves show a constructive interference. Here, averaging over $\delta t_{1,2}$

$$I \propto \frac{1}{2} \left(1 + e^{-\omega^2 \tau L/c}\right)$$

For large $\boxed{L, t}$ the intensity behaves as the two waves *were not interfering*

The relevant parameter here is $\boxed{\omega^2 \tau L/c \geq 1}$.

A preliminary analysis puts for Virgo, whose sensibility is bound by the shot noise, a bound

$$\boxed{\tau < 10^{-10} \mathrm{GeV}^{-1} \hbar}$$

The harmonic oscillator can be treated in a similar way, as a system with variable mass $M = m/(1+\varepsilon)$ and frequency $\Omega = \omega(1+\varepsilon)$ with $M\Omega = m\omega$ constant.

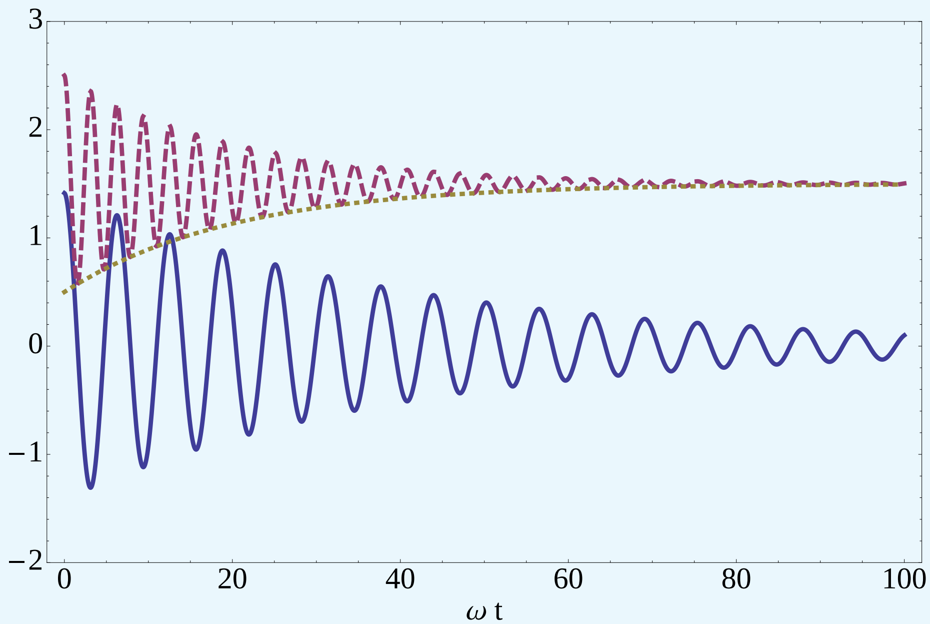
This can be solved using standard techniques (Dyson series) so that the time evolution operator can be formally integrated

$$\overline{\mathbf{a}(t)} = \mathbf{a}(0)e^{-i\,\omega t}\sum_k \frac{(-\omega^2\tau t)^k}{2^k k!} = \mathbf{a}(0)e^{-i\,\omega t}e^{-\omega^2\tau t/2}$$

Apart from standard oscillatory term, evolution is exponentially damped on time–scales larger than the characteristic time $2(\omega^2\tau)^{-1}$

This can be applied to coherent states

The time evolution of position mean value $\overline{\langle \mathbf{x} \rangle}_\lambda$ (solid), $\overline{\langle \mathbf{x}^2 \rangle}_\lambda$ (long-dashed) and squared uncertainty $\overline{\Delta \mathbf{x}^2}_\lambda$ (short-dashed) for $\lambda = 1$ in units of appropriate powers of $\sqrt{\hbar/(m\omega)}$.



We choose an unrealistic large value $\omega\tau = 0.05$

This behavior can be translated in the capability of an optical coherent state to survive in a cavity

The effect of the variable $\boxed{\hbar}$ is that the state decoherentizes.

This must be compared with the fact that in any case coherent states in real cavities do not last forever. The two competing effects are however different for scales and functional dependence on time

Present technology, without dedicated experiments, give an order of magnitude for the bound to be

$$\boxed{\tau < 10^{-8}\mathrm{GeV}^{-1}\hbar}$$

Let me now consider the case in which the inconstant constant is Newton's gravitational constant $\boxed{G_{\mathsf{N}}}$

Assume that is rapidly and stochastically oscillating around its average:

$$\boxed{G_{\mathsf{N}}(t) = \overline{G}_{\mathsf{N}}(1 + \sigma\xi(t))}$$

Newton's gravitational constant appears in the Einstein equation:

$$\boxed{G_{ab} = 8\pi G_{\mathsf{N}}\, T_{ab}}$$

Turning G_{N} naively into a dynamical variable, whith T_{ab} covariantly conserved, violates the Bianchi identities

$$0 = \nabla^a G_{ab} = 8\pi \nabla^a (G_{\mathrm{N}} T_{ab})$$

The solution to the impasse is to split the energy-momentum tensor into a matter term, which satisfies the usual conservation law, and a correction term such that Bianchi is satisfied:

$$T_{ab} = T_{ab}^{\mathrm{matter}} + \tau_{ab}$$

We therefore mimic the effect of a variable G_{N} with the presence of a fictitious gravitational source.

In a isotropic and homogeneous cosmological model, we have

$$T_{ab}^{\text{matter}} = (\rho + p)u_a u_b + p g_{ab}$$

where u_a is tangent to the geodesic of a isotropic observer and the fluid satisfies the equation of state $p = w\rho$ and the continuity equation $\dot{\rho} + 3H(\rho + p) = 0$.

For the extra term it is natural to assume that (at least in average) it is some form of "dark energy"

$$\tau_{ab} = -\lambda g_{ab}$$

For simplicity consider a toy model for which the only variation is in time.

The evolution of the time-dependent "cosmological constant" follows from the Bianchi identities $\dot{G}_{\mathsf{N}}(\rho + \lambda) + G_{\mathsf{N}}\dot{\lambda} = 0$

After some substitutions, and assuming that we are in a matter or radiation dominated era ($\lambda(t_i) \ll \rho(t_i)$), we get a deformed equation for the evolution of the universe

$$H^2(t) = \frac{8\pi}{3}\left(G_{\mathsf{N}}(t_i)\rho(t_i) + \int_{t_i}^{t} G_{\mathsf{N}}\dot{\rho}\right)$$

with H the Hubble parameter which describes the expansion of the universe after the big bang. Note that all dark energy contributions are in the variation of G_N

Differentiating the we get the evolution equation:

$$\dot{H} = -4\pi G_{\mathsf{N}}\rho$$

With the constraints on the initial conditions $H^2(t_i) = \frac{8\pi}{3}G_{\mathsf{N}}(t_i)\rho(t_i)$

In analogy with the previous case we consider:

$$G_{\mathsf{N}}(t) = \overline{G}_{\mathsf{N}}(1 + \sigma\xi(t))$$

$$\langle\xi(t)\rangle = 0, \quad ; \quad \langle\xi(t)\xi(t')\rangle = \delta(t - t')$$

Rewriting the evolution equation in a differential form and assuming that $\xi(t)$ has a white noise distribution we get

$$\mathrm{d}H = -4\pi\overline{G}_{\mathsf{N}}\rho(\mathrm{d}t + \sigma\mathrm{d}W_t)$$

where W_t is a Wiener process. Loosely speaking the white noise is the derivative of the Wiener process

The Wiener process W_t has the following properties:

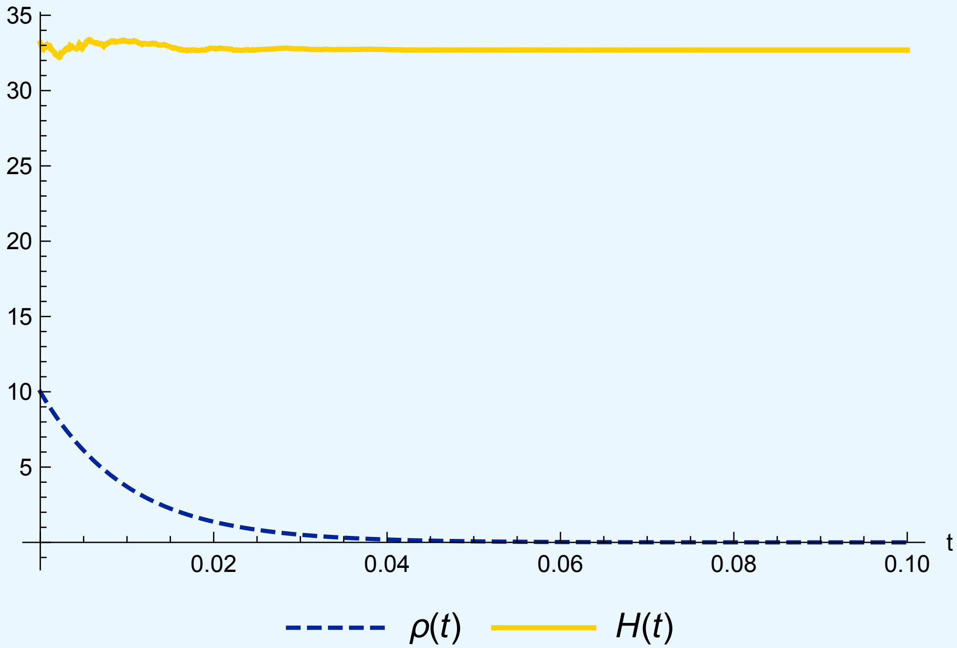
- $\boxed{W_{t_i} = 0}$ with probability $\boxed{1}$;

- The increments $\boxed{W_{t_{k+1}} - W_{t_k}}$ are statistically independent Gaussian variables with mean $\boxed{o}$ and variance $\boxed{t_{k+1} - t_k = h}$.

It plays the role of a stochastic driving term

We have investigated numerically the solutions for such a model with parameters $\boxed{0 \leq \sigma \leq 1}$, $\boxed{10^{-6} \leq h \leq 10^{-2}}$, in units such that $\boxed{\overline{G}_N = 1}$, for different choices of the initial condition $\boxed{\rho(t_i)}$.

Qualitatively the behaviour of the solutions is the same



The parameters are $\boxed{\sigma = 0.1}$, $\boxed{\rho_0 = 10}$, $\boxed{h = 10^{-4}}$, The horizontal asymptote of $\boxed{H(t)}$ to a late era of accelerated expansion, when the Universe is dominated by a "cosmological constant". The qualitative behaviour of the solutions is a general feature, which does not depend on the particular choice of parameters.

As $t \to \infty$, ρ is standard , while H attains a nonvanishing positive limit. The value of the asymptotic limit does not vary much with the random sequence, and is not particularly sensitive to the parameters.

This is a *general feature of the model* that does not depend on the particular values chosen for ρ_0 or the noise strength σ. For $\sigma = 0$ one recovers the standard cosmology in the matter-dominated era, namely $\rho \propto 1/t^2$ and $H \propto 1/t$.

We did not put "numbers" in the model, as it is too rough at present, but what we find important is the fact that stochastic variations can stabilize a cosmological constant to a nonzero value

Conclusions

I think that the possibility to have some generic “model independent” consequences of quantum space time investigated is an important activity that our community should pursue.

On one side it helps keep *our feet on the ground*, and on the other side it may stimulate some also some very interesting mathematical structures. For example what is mathematically a space with microspoc random fluctuations in the metric?

And, who knows, it may also have something to do with the real world!