Quantum spacetime, the view from below

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I think we would all agree for the need for fundamental new physics at very high energy

If we want to describe the big bang we will need some sort of quantum gravity, of which we have several flavours presently on the market

The paradigm I will follow for this talk is that at very high scale there is a new form of geometry, and to describe it I will use the tools of Noncommutative Geometry

Please note that here by noncommutative I do not mean a space in which the coordinates do not commute like $\boxed{[x^\mu, x^\nu] = i\theta^{\mu\nu}}$. I mean the algebraic tools given by spectral geometry

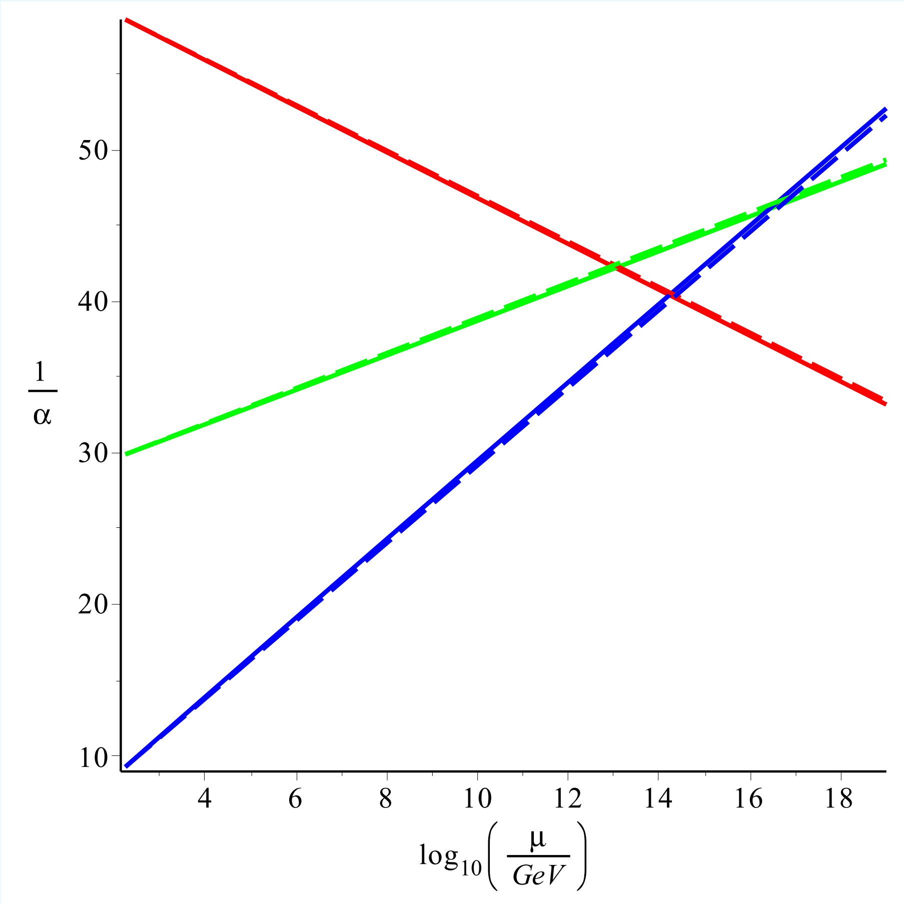
I will also try to learn what we can learn "looking from below"

I will use the knowledge form field theory at energies below the (yet to be defined) transition scale at which quantum geometry appears , to infer some knowledge of quantum spacetime

We are having new data from “high” energy experiments, mainly LHC, so this is a good moment to explore the consequences of field theory.

The way one can learn what happens beyond the scale of an experiment is to use the renormalization flow of the theory

We know that the coupling constants, i.e. the strength of the interaction, change with energy.



This picture is valid in the absence of new physics, i.e. new particles and new interactions which would alter the equations which govern the running

The three interaction strength start from rather different values but come together almost at a single unification point

But then the nonabelian interactionsprocced towards asymptotic freedom, while the abelian one climbs towards a Landau pole at incredibly high energies 10^{53} GeV

The lack of a unification point was one of the reasons for the falling out of fashion of GUT's.

Some supersymmetric theories have unification point

We all believe that this running will be stopped by "something" at 10^{19} GeV

This unknown something we call quantum gravity

As I said I take the point of view that there is a fundamental change of the degrees of freedom of spacetime at, or before the Planck scale, and that the tools to describe this are the one of Noncomutative Spectral Geometry

Following this paradigm, for me the topological nature of space-time is in the spectrum of the algebra of functions on spacetime (or its generalization)

The metric and geometric properties are encoded in the (generalized) Dirac operator D which fixes the background around which expand the action

The eigenvalues of the Dirac operator on a curved spacetime are diffeomorphism-invariant functions of the geometry. They form an infinite set of observables for general relativity.

The interaction among fields is described by the Chamseddine-Connes Spectral Action

$$S = \mathrm{Tr}\,\chi\left(\frac{D_A^2}{\Lambda^2}\right)$$

χ is a cutoff function, which we may take to be a decreasing exponential or the characteristic function of the interval: $\chi(x) = \begin{cases} 1 & x \leq 1 \\ 0 & x > 1 \end{cases}$

$D_A = D + A$ is a fluctuation of the Dirac operator, A a connection one-form built from D as $A = \sum_i a_i[D, b_i]$ with a, b elements of the algebra, the fluctuations are ultimately the variables, the fields of the action

Λ is a cutoff scale

The spectral action can be expanded in powers of $\boxed{\Lambda^{-1}}$ using standard heat kernel techniques

In this framework it is possible to describe the action of the standard model

One has to choose as $\boxed{D}$ operator the tensor product of the usual Dirac operator on a curved background $\boxed{\nabla\!\!\!\!/}$ times a matrix containing the fermionic parameters of the standard model (Yukawa couplings and mixings), acting on the Hilbert space of fermions

In this way one "saves" one parameter, and can predict the mass of the Higgs. The original prediction was $\boxed{\text{170 GeV}}$, which is not a bad result considering that the theory is basically based on pure mathematical requirements

When it was found at $\boxed{\text{125 GeV}}$ it was realized that the model was refined (right handed neutrinos play a central role) to make it compatible with present experiments. (Stephan, Devastato Martinetti, FL, Chamseddine, Connes, Van Suijlekom But this is yesterday Perimeter's seminar...

Technically the bosonic spectral action is a sum of residues and can be expanded in a power series in terms of Λ^{-1} as

$$S_B = \sum_n f_n \, a_n (D_A^2/\Lambda^2)$$

where the f_n are the momenta of χ

$$\begin{aligned} f_0 &= \int_0^\infty \mathrm{d}x \, x\chi(x) \\ f_2 &= \int_0^\infty \mathrm{d}x \, \chi(x) \\ f_{2n+4} &= (-1)^n \partial_x^n \chi(x) \Big|_{x=0} \quad n \geq 0 \end{aligned}$$

the a_n are the Seeley-de Witt coefficients which vanish for n odd. For D_A^2 of the form

$$D^2 = -(g^{\mu\nu}\partial_\mu\partial_\nu \mathbb{I} + \alpha^\mu \partial_\mu + \beta)$$

Defining (in term of a generalized spin connection containing also the gauge fields)

$$
\begin{aligned}
\omega_\mu &= \frac{1}{2} g_{\mu\nu} \left(\alpha^\nu + g^{\sigma\rho}\Gamma^\nu_{\sigma\rho}\mathbb{I}\right) \\
\Omega_{\mu\nu} &= \partial_\mu\omega_\nu - \partial_\nu\omega_\mu + [\omega_\mu, \omega_\nu] \\
E &= \beta - g^{\mu\nu}\left(\partial_\mu\omega_\nu + \omega_\mu\omega_\nu - \Gamma^\rho_{\mu\nu}\omega_\rho\right)
\end{aligned}
$$

then

$$
\begin{aligned}
a_0 &= \frac{\Lambda^4}{16\pi^2}\int \mathrm{d}x^4\sqrt{g}\,\mathrm{tr}\,\mathbb{I}_F \\
a_2 &= \frac{\Lambda^2}{16\pi^2}\int \mathrm{d}x^4\sqrt{g}\,\mathrm{tr}\left(-\frac{R}{6} + E\right) \\
a_4 &= \frac{1}{16\pi^2}\frac{1}{360}\int \mathrm{d}x^4\sqrt{g}\,\mathrm{tr}\,(-12\nabla^\mu\nabla_\mu R + 5R^2 - 2R_{\mu\nu}R^{\mu\nu} \\
&\quad +2R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} - 60RE + 180E^2 + 60\nabla^\mu\nabla_\mu E + 30\Omega_{\mu\nu}\Omega^{\mu\nu})
\end{aligned}
$$

tr is the trace over the inner indices of the finite algebra $\mathcal{A}_F$ and Ω and E contain the gauge degrees of freedom including the gauge stress energy tensors and the Higgs, which is given by the inner fluctuations of D

Let me dwell on the role of Λ. The spectral action uses it as a regulator. Without it, the trace diverges.

This points to a geometry in which the Dirac operator is actually a truncated operator, i.e. the Dirac operator has a spectrum of increasing eigenvalues which "saturates" at Λ

Consider the eigenvectors $|n\rangle$ of D in increasing order of the respective eigenvalue λ_n.

Call N the number of eigenvalues smaller then Λ, i.e. $\lambda_n < \Lambda$ for $n < N$, zero otherwise.

Truncation of the spectrum can be made by the projection operator

$$P_\Lambda = \sum_{n \le N} |n\rangle \langle n|$$

Defining

$$D_\Lambda = D P_\Lambda + (1 - P_\Lambda)\,\Lambda, \quad P_\Lambda = \Theta(\Lambda^2 - D^2)$$

We are effectively saturating the operator at a scale $\boxed{\Lambda}$

Given a space with a Dirac operator one can define a distance (Connes) between states of the algebra of functions, in particular points are (pure) states and the distance is:

$$d(x,y) = \sup_{\|[D,f]\| \leq 1} |f(x) - f(y)|$$

It is possible possible to prove D'Andrea, FL, Martinetti that using D_Λ the distance among points is infinite

In general for a bounded Dirac operator of norm Λ then $d(,x,y) > \Lambda^{-1}$, and to find states at finite distance one has to consider "extended" points, such as coherent spates

The role of a truncated Dirac operator, or in general wave operator, has been introduced in field theory even before the sacral action. It goes under the name of Finite Mode Regularization,

Andrianov, Bonora, Fujikawa

Consider the generic fermionic action:

$$\boxed{Z = \int [d\bar{\psi}][d\psi] e^{-\langle\psi|D|\psi\rangle} \overset{\text{formally}}{=} \det D}$$

The equality is formal because the expression is divergent, and has to be regularized, for example considering $\boxed{D_\Lambda}$

One can study the renormalization flow, and note that the measure is not invariant under scale transformation, giving rise to a potential anomaly Andrianov, Kurkov, FL

The induced term by the flow, which takes care of the anomaly, turn out to be exactly the spectral action

The hypothesis is that $\boxed{\Lambda}$ has a physical meaning, it is a scale indicating a phase transition, and we can try to infer some properties of the phase above $\boxed{\Lambda}$ studying the high energy limit of the action with the cutoff.

At high momentum Green's function, the inverse of $\boxed{D_\Lambda}$, effectively is the identity in momentum space

I will now see this in greater detail considering the bosonic sector

Usually probes are bosons, hence let me consider the expansion of the spectral action in the high momentum limit Kurkov, FL, Vassilevich

This has been made by Barvinsky and Vilkovisky who were able to sum all derivatives (for a decreasing exponential cutoff function):

$$\mathrm{Tr}\,\exp\left(-\frac{D^2}{\Lambda^2}\right) \simeq \frac{\Lambda^4}{(4\pi)^2}\int d^4x\sqrt{g}\,\mathrm{tr}\,\Big[1+\Lambda^{-2}P+$$

$$\Lambda^{-4}\left(R_{\mu\nu}f_1\left(-\frac{\nabla^2}{\Lambda^2}\right)R^{\mu\nu}+Rf_2\left(-\frac{\nabla^2}{\Lambda^2}\right)R+\right.$$

$$\left.Pf_3\left(-\frac{\nabla^2}{\Lambda^2}\right)R+Pf_4\left(-\frac{\nabla^2}{\Lambda^2}\right)P+\Omega_{\mu\nu}f_5\left(-\frac{\nabla^2}{\Lambda^2}\right)\Omega^{\mu\nu}\right)\Big]+O(R^3,\Omega^3,E^3)$$

where $P = E + \frac{1}{6}R$ and $f_1, \ldots, f_5$ are known functions, high momenta asymptotic of form factor:

$f_1 \ldots f_5$ read:

$$
\begin{aligned}
f_1(\xi) &\simeq \frac{1}{6}\xi^{-1} - \xi^{-2} + O\left(\xi^{-3}\right) \\
f_2(\xi) &\simeq -\frac{1}{18}\xi^{-1} + \frac{2}{9}\xi^{-2} + O\left(\xi^{-3}\right) \\
f_3(\xi) &\simeq -\frac{1}{3}\xi^{-1} + \frac{4}{3}\xi^{-2} + O\left(\xi^{-3}\right) \\
f_4(\xi) &\simeq \xi^{-1} + 2\,\xi^{-2} + O\left(\xi^{-3}\right) \\
f_5(\xi) &\simeq \frac{1}{2}\xi^{-1} - \xi^{-2} + O\left(\xi^{-3}\right)
\end{aligned}
$$

Let me consider a Dirac operator containing just the relevant aspects, i.e. a bosonic fields and the fluctuations of the metric.

$$D\!\!\!\!/ = i\gamma^\mu \nabla_\mu + \gamma_5 \phi = i\gamma^\mu(\partial_\mu + \omega_\mu + iA_\mu) + \gamma_5 \phi$$

with ω_μ the Levi-Civita connection and $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$

It is now possible to perform the B-V expansion to get the expression for the high energy spectral action

$$S_B \simeq \frac{\Lambda^4}{(4\pi)^2} \int d^4x \left[-\tfrac{3}{2} h_{\mu\nu} h_{\mu\nu} + 8\phi \frac{1}{-\partial^2} \phi + 8 F_{\mu\nu} \frac{1}{(-\partial^2)^2} F_{\mu\nu} \right]$$

In order to understand the meaning of this action let me remind how we get propagation of waves and correlation of points in usual QFT with action

$$S[J,\varphi] = \int d^4x \left[\varphi(x)\left(\partial^2 + m^2\right)\varphi(x) - J(x)\varphi(x)\right]$$

To this correspond the equation of motion

$$\left(\partial^2 + m^2\right)\varphi(y) = J(y)$$

And the Green's function $G(x-y)$ which "propagates" the source:

$$\varphi_J(x) = \int d^4y J(y) G(x-y)$$

In momentum representation we have

$$\varphi(x) = \frac{1}{(2\pi)^2} \int d^4k \ e^{ikx} \ \widehat{\varphi}(k)$$

$$J(x) = \frac{1}{(2\pi)^2} \int d^4k \ e^{ikx} \ \hat{J}(k)$$

$$G(x-y) = \frac{1}{(2\pi)^2} \int d^4k \ e^{ik(x-y)} \ \widehat{G}(k)$$

And the propagator is

$$G(k) = \frac{1}{\left(k^2 + m^2\right)}$$

The field at a point depends on the value of field in nearby points, and the points “talk” to each other exchanging virtual particles

In the general case of a generic boson φ, the Higgs, an intermediate vector boson or the graviton

$$S[J,\phi] = \int d^4x \left(\frac{1}{2}\varphi(x)F(\partial^2)\varphi(x) - J(x)\varphi(x)\right) ,$$

In this case the equation of motion is $F(\partial^2)\phi(x) = J(x)$ giving

$$G = \frac{1}{F(\partial^2)} \quad , \quad G(k) = \frac{1}{F(-k^2)}$$

and

$$\varphi_J(x) = \int d^4y J(y) G(x-y) = \frac{1}{(2\pi)^4} \int d^4k e^{ikx} J(k) \frac{1}{F(-k^2)}$$

The cutoff is telling us that

$$\varphi_J(x) = \int d^4y J(y) G(x-y) = \frac{1}{(2\pi)^4} \int d^4k e^{ikx} J(k) \frac{1}{F(-k^2)}$$

The short distance behaviour is given by the limit $k \to \infty$

Consider $J(k) \neq 0$ for $|k^2| \in [K^2, K^2 + \delta k^2]$, with K^2 very large.

$$\varphi_J(x) \xrightarrow[K\to\infty]{} \begin{cases} \frac{1}{(2\pi)^4} \int d^k e^{ikx} J(k) k^2 = (-\partial^2) J(x) & \text{for scalars and vectors} \\ \frac{1}{(2\pi)^4} \int d^k e^{ikx} J(k) = J(x) & \text{for gravitons} \end{cases}$$

This corresponds to a limit of the Green's function in position space

$$G(x-y) \propto \begin{cases} (-\partial^2)\delta\,(x-y) & \text{for scalars and vectors} \\ \delta\,(x-y) & \text{for gravitons} \end{cases}$$

The correlation vanishes for noncoinciding points, heuristically, nearby points "do not talk to each other".

This is a limiting behaviour, I think one has to take it as a general indication that the presence of a physical cutoff scale in momenta leads to a "non geometric phase" in which the concept of point ceases to have meaning, possibly described by a noncommutative geometry

Note that throughout this discussion I have done nothing to spacetime, I have only imposed the cutoff and used standard techniques and interpretations

Conclusions

It is very difficult to explain to a deep water fish the concept of *air*

Still he will know that as he goes up the pressure will decrease

And he can also infer some properties of a different states of matter by looking at bubbles which are creates near some “high energy” volcanic vents or when “above” there are storms

What I presented here is hopefully just an element of a larger picture which should use gravity in a fundamental way

I am convinced that what has to change at high scales is the very geometry of spacetime, and this work is but an indication that the direction is the loss of correlation among the points