Grand Symmetry, Spectral Action and the Higgs Mass

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One of the nice aspects of the NCG approach to the standard model is that the constrains of the "internal geometry" almost single out the symmetries

The requirements that the finite dimensional internal space be a noncommutative manifold, together with the first order condition, give a well definite pattern to the symmetries of the theory. Once the Hilbert space of known fermions is given

There is not much more one can do without enlarging the Hilbert space

In particular no usual Grand Unified Theory, like $SU(5)$ or $SO(10)$, can be achieved. This because algebras have less representations than groups. In finite dimensions they only have one nontrivial representation.

In this talk I will propose however the existence of a larger symmetry, which I will call *Grand Symmetry*. This symmetry mixes internal and spin indices, and is therefore some sort of pregeometric symmetry, relating to a phase prior to the emergence of the symmetries of spacetime.

We will also see how the field $\boxed{\sigma}$, which is necessary for the correct mass of the Higgs, is a consequence of the presence of right-handed neutrinos with Majorana mass

The grand algebra allows the presence of $\boxed{\sigma}$ without the violation of the first order condition, which is preserved all along, and also allows for an explanation of the size of a high mass for the right neutrino, which is necessary for the see-saw mechanism.

We are considering almost commutative geometry, the algebra is of the kind

$$\mathcal{A} = C^\infty(\mathcal{M}) \otimes \mathcal{A}_F$$

We will consider various finite dimensional matrix algebras, but throughout the whole talk the Hilbert space will always be

$$\mathcal{H} = sp(L^2(\mathcal{M})) \otimes \mathcal{H}_F$$

where $sp(L^2(\mathcal{M}))$ are square summable spinors and

$$\mathcal{H}_F = \mathcal{H}_R \oplus \mathcal{H}_L \oplus \mathcal{H}_R^c \oplus \mathcal{H}_L^c = \mathbb{C}^{96}$$

contains all the 96 particle-degrees of freedom of the standard model

(8 fermions, with two chiralities, plus antiparticles per three generations $8 \times 2 \times 2 \times 3 = 96$

Also the chiral and real structure will be the same for the whole paper and are

$$\Gamma = \gamma^5 \otimes \gamma_F, \qquad J = \mathcal{J} \otimes J_F$$

with $\mathcal{J}$ the charge conjugation operator and

$$\gamma_F = \begin{pmatrix} \mathbb{I}_{8N} & & & \\ & -\mathbb{I}_{8N} & & \\ & & -\mathbb{I}_{8N} & \\ & & & \mathbb{I}_{8N} \end{pmatrix}, \qquad J_F = \begin{pmatrix} 0 & \mathbb{I}_{16N} \\ \mathbb{I}_{16N} & 0 \end{pmatrix} cc$$

with cc the complex conjugation

The Dirac operator contains all information about masses and Yukawa coupling. I will not write it explicitly at this stage.

There are several finite dimensional algebras in this game, and I want to look at their representations

Ultimately we want to ge to the the standard model algebra

$$\boxed{\mathcal{A}_{sm} = \mathbb{C} \oplus \mathbb{H} \oplus \mathbb{M}_3(\mathbb{C}),}$$

$\boxed{\mathbb{H}}$ are the quaternions, which we represent as $\boxed{2 \times 2}$ matrices

It is possible to have this emerge from the most general algebra which satisfies the condition of being a noncommutative manifold

$$\boxed{\mathcal{A}_{\mathcal{F}} = \mathbb{M}_a(\mathbb{H}) \oplus \mathbb{M}_{2a}(\mathbb{C}) \qquad a \in \mathbb{N}.}$$

This algebra acts on a finite Hilbert space of dimension $\boxed{2(2a)^2}$. For a non trivial grading it must be $\boxed{a \leq 2}$

$$\boxed{\mathcal{A}_F = \mathbb{M}_2(\mathbb{H}) \oplus \mathbb{M}_4(\mathbb{C})}$$

Hence an Hilbert space of dimension $\boxed{2(2 \cdot 2)^2 = 32}$, the dimension of $\boxed{\mathcal{H}_F}$ for one generation.

The grading condition $\boxed{[a, \Gamma] = 0}$ reduces the algebra to the left-right algebra

$$\boxed{\mathcal{A}_{LR} = \mathbb{H}_L \oplus \mathbb{H}_R \oplus \mathbb{M}_4(\mathbb{C})}$$

The order one condition reduces further the algebra to $\mathcal{A}_{sm}$, i.e. the algebra whose unimodular group is U(1)×SU(2)×U(3)

Let us look in detail to a vector in the Hilbert space:

$$\Psi^{\mathsf{C}\mathrm{I}m}_{s\dot{s}\alpha}(x) \in \mathcal{H} = L^2(\mathcal{M}) \otimes \mathsf{H}_F = sp(L^2(\mathcal{M})) \otimes \mathcal{H}_F$$

Note the difference between $\mathcal{H}_F$, which is 96 dimensional, and H_F which is 384 dimensional. The meaning of the indices is as follows:

$$\Psi^{\mathsf{C}\mathrm{I}m}_{s\dot{s}\alpha}(x)$$

$$\begin{array}{c} s = r, l \\ \dot{s} = \dot{0}, \dot{1} \end{array}$$

are the spinor indices. They are not internal indices in the sense that the algebra $\mathcal{A}_F$ acts diagonally on it. They take two values each, and together they make the four indices on an ordinary Dirac spinor.

$$\Psi^{\mathsf{c}\mathrm{I}m}_{s\dot{s}\alpha}(x)$$

$\mathrm{I} = 0, \ldots 3$ indicates a "lepto-colour" index. The zeroth "colour" actually identifies leptons while $\mathrm{I} = 1, 2, 3$ are the usual three colours of QCD.

$$\Psi^{\mathsf{c}\mathrm{I}m}_{s\dot{s}\alpha}(x)$$

$\alpha = 1 \ldots 4$ is the flavour index. It runs over the set u_R, d_R, u_L, d_L when $\mathrm{I} = 1, 2, 3$, and ν_R, e_R, ν_L, e_L when $\mathrm{I} = 0$. It repeats in the obvious way for the other generations.

$$\boxed{\Psi^{\mathsf{C}\mathrm{I}m}_{s\dot{s}\alpha}(x)}$$

$\boxed{\mathsf{C} = 0, 1}$ indicates whether we are considering "particles" ($\boxed{\mathsf{C} = 0}$) or "antiparticles" ($\boxed{\mathsf{C} = 1}$).

$$\boxed{\Psi^{\mathsf{c}\mathrm{I}m}_{s\dot{s}\alpha}(x)}$$

$\boxed{m = 1, 2, 3}$ is the generation index. The representation of the algebra of the standard model is diagonal in these indices, the Dirac operator is not, due to Cabibbo-Kobayashi-Maskawa mixing parameters. For this seminar plays no role, and will ignored.

We can now give explicitly the algebra representations in term of these indices.

We start from $\mathcal{A}_F = \mathbb{M}_2(\mathbb{H}) \oplus \mathbb{M}_4(\mathbb{C})$, a generic element will depend on 4×4 complex matrix m, and a 2×2 matrix of quaternions q , which we may also see as a 4×4 with some conditions

The representation in its fullness is

$$A_{s\dot{s}\mathsf{D}\mathsf{J}\alpha}^{t\dot{t}\,\mathsf{C}\mathsf{I}\beta} = \delta_s^t \delta_{\dot{s}}^{\dot{t}} \left(\delta_0^{\mathsf{C}} \delta_{\mathsf{J}}^{\mathsf{I}} Q_\alpha^\beta + \delta_1^{\mathsf{C}} M_{\mathsf{J}}^{\mathsf{I}} \delta_\alpha^\beta \right)$$

Note the two δ's at the beginning which show that the algebra acts trivially on the spacetime indices, and the fact that the two matrices act on different indices. This ensures the order zero condition, namely exchanging particles with antiparticles, the job done by J, the two representations commute.

The representations of the other algebra are similar, in the case of the standard model there is a differentiation with the leptocolour indices.

The order one condition and a $\boxed{\nu}$ Majorana mass cause the reduction to $\boxed{C^\infty(\mathcal{M}) \otimes \mathcal{A}_{sm}}$, represented as

$\boxed{a = \{m, q, c\} \text{ with } m \in C^\infty(\mathcal{M}) \otimes \mathbb{M}_3(\mathbb{C}),\ q \in C^\infty(\mathcal{M}) \otimes \mathbb{H},\ c \in C^\infty(\mathcal{M}) \otimes \mathbb{C}}$

is

$$\boxed{a^{t\dot{t}\mathsf{CI}\beta}_{s\dot{s}\mathsf{DJ}\alpha} = \delta^t_s \delta^{\dot{t}}_{\dot{s}} \left(\delta^{\mathsf{C}}_0 \delta^{\mathsf{I}}_{\mathsf{J}} \left(\mathsf{q}^\beta_\alpha + \mathsf{c}^\beta_\alpha \right) + \delta^{\mathsf{C}}_1 \left(\mathsf{m}^{\mathsf{I}}_{\mathsf{J}} + \tilde{\mathsf{c}}^{\mathsf{I}}_{\mathsf{J}} \right) \delta^\beta_\alpha \right)}$$

where we use the following $\boxed{4 \times 4}$ complex matrices:

$$\mathsf{q} = \begin{pmatrix} 0_2 & \\ & q \end{pmatrix}_{\alpha\beta}, \qquad \mathsf{c} = \begin{pmatrix} c & & \\ & \bar{c} & \\ & & 0_2 \end{pmatrix}_{\alpha\beta}, \qquad \tilde{\mathsf{c}} = \begin{pmatrix} c & \\ & 0_3 \end{pmatrix}_{\mathsf{IJ}}, \qquad \mathsf{m} = \begin{pmatrix} 0 & \\ & m \end{pmatrix}_{\mathsf{IJ}}$$

The breaking from $\mathcal{A}_F$ to $\mathcal{A}_{sm}$ goes with the chirality and first order conditions

I can similarly write down the Dirac operator

$$D = \partial\!\!\!/ \otimes \mathbb{I}_{96} + \gamma^5 \otimes D_F$$

$$D_F = \begin{pmatrix} 0_{8N} & \mathcal{M} & \mathcal{M}_R & 0_{8N} \\ \mathcal{M}^\dagger & 0_{8N} & 0_{8N} & 0_{8N} \\ \mathcal{M}_R^\dagger & 0_{8N} & 0_{8N} & \bar{\mathcal{M}} \\ 0_{8N} & 0_{8N} & \mathcal{M}^T & 0_{8N} \end{pmatrix}.$$

$\mathcal{M}$ contains the Dirac-Yukawa couplings. It links left with right particles. $\mathcal{M}_R = \mathcal{M}_R^T$ contains Majorana masses and links righr particles with right antiparticles. $\mathcal{M} = \begin{pmatrix} M_u & 0_{4N} \\ 0_{4N} & M_d \end{pmatrix} \qquad \mathcal{M}_R = \begin{pmatrix} M_R & 0_{4N} \\ 0_{4N} & 0_{4N} \end{pmatrix}$ where M_u containins the masses of the up, charm and top quarks and the neutrinos (Dirac mass), M_R contains the Majorana neutrinos masses and M_d the remaining quarks and electrons, muon and tau masses, including mixings

I think by now you know the rules. With the algebra and D one builds the one-form, which are the fluctuations of the Dirac operator.The bosonic fields are coming from these one-form $\sum_i a_i[D, b_i]$

But here we run into a problem: the elements of $\mathcal{M}_R$ are the ones which should give rise to the field σ as intermediate boson, on a par with the Higgs, and relate to the breaking of the left-right symmetry.

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Or we may look for a bigger algebra...

Consider the case of $\mathbb{M}_a(\mathbb{H}) \oplus \mathbb{M}_{2a}(\mathbb{C})$ for the case $a = 4$

In this case we need a $2 \cdot (2 \cdot 4)^2 = 128$ dimensional space, which for 3 generations gives a 384 dimensional Hilbert space.

I need a representation of the algebra $\mathbb{M}_4(\mathbb{H}) \oplus \mathbb{M}_8(\mathbb{C})$ acting on the spinors I gave earlier, and which can satisfy the stringent order zero conditions

Consider $Q \in \mathbb{M}_4(\mathbb{H})$ and $M \in \mathbb{M}_8(\mathbb{C})$ with indices

$$Q^{\dot{t}\beta}_{\dot{s}\alpha} = \begin{pmatrix} Q^{\dot{0}\beta}_{\dot{0}\alpha} & Q^{\dot{1}\beta}_{\dot{0}\alpha} \\ Q^{\dot{0}\beta}_{\dot{1}\alpha} & Q^{\dot{1}\beta}_{\dot{1}\alpha} \end{pmatrix}_{\dot{s}\dot{t}}, \qquad M^{t\mathrm{I}}_{s\mathrm{J}} = \begin{pmatrix} M^{r\mathrm{I}}_{r\mathrm{J}} & M^{l\mathrm{I}}_{r\mathrm{J}} \\ M^{r\mathrm{I}}_{l\mathrm{J}} & M^{l\mathrm{I}}_{l\mathrm{J}} \end{pmatrix}_{st}$$

where, for any $\dot{s}, \dot{t} \in \{\dot{0}, \dot{1}\}$ and $s, t \in \{l, r\}$, the matrices $Q^{\dot{t}\beta}_{\dot{s}\alpha} \in \mathbb{M}_2(\mathbb{H}), \quad M^{t\mathrm{I}}_{s\mathrm{J}} \in \mathbb{M}_4(\mathbb{C})$ have the index structure defined above

The representation of the element $\boxed{A = (Q, M) \in \mathcal{A}_G}$ is:

$$\boxed{\mathsf{A}^{t\dot{t}\mathsf{C}\,\mathsf{I}\beta}_{s\dot{s}\mathsf{D}\mathsf{J}\alpha} = \left(\delta^{\mathsf{C}}_{\mathsf{0}}\delta^{t}_{s}\delta^{\mathsf{I}}_{\mathsf{J}}\, Q^{\dot{t}\beta}_{\dot{s}\alpha} + \delta^{\mathsf{C}}_{\mathsf{1}} M^{t\mathsf{I}}_{s\mathsf{J}}\delta^{\dot{t}}_{\dot{s}}\delta^{\beta}_{\alpha}\right)}$$

compare with the previous case

$$\boxed{A^{t\dot{t}\,\mathsf{C}\mathsf{I}\beta}_{s\dot{s}\mathsf{D}\mathsf{J}\alpha} = \delta^{t}_{s}\delta^{\dot{t}}_{\dot{s}}\left(\delta^{\mathsf{C}}_{\mathsf{0}}\delta^{\mathsf{I}}_{\mathsf{J}} Q^{\beta}_{\alpha} + \delta^{\mathsf{C}}_{\mathsf{1}} M^{\mathsf{I}}_{\mathsf{J}}\delta^{\beta}_{\alpha}\right)}$$

The spinor indices and the internal gauge indices are mixed. We are in a phase in which the Euclidean structure of space time has not yet emerged.

The fermions are not yet fermions

We envisage this Grand Symmetry to belong to a pre geometric phase. At this stage all elements of D_F may be negligible, and the sponsorial part of the direct operator $\partial\!\!\!/$ will cause the "breaking" to a phase in which the symmetries of the phase space emerge

In particular, the order one condition for $\partial\!\!\!/$ causes the reduction of the algebra to $\mathbb{M}_2(\mathbb{H}) \oplus \mathbb{M}_4(\mathbb{C})$

And there is an added bonus:

This grand algebra, and a corresponding $\boxed{D}$ operator, have "more room" to operate. Although the Hilbert space is the same, the fact that we abandoned the factorization of the internal indices, gives us more entries to accommodate the Majorana masses

Hence we can put a Majorana mass for the neutrino and at the same time satisfy the order one condition. Then the one form corresponding to this $\boxed{D_\nu}$ will give us the by now famous field $\boxed{\sigma}$, which can only appear before the transition to the geometric spacetime

The natural scale for this mass is to be above a transition which gives the geometric structure. Therefore it is natural that it may be at a high scale. How high we can discuss

The grand symmetry is no ordinary gauge symmetry, there is never a $\boxed{SU(8)}$ in the game for example

It represents a phase in which the internal noncommutative geometry contains also the spin structure, even the Lorentz (Euclidean) structure of space time in a mixed way

The differentiation between the spin structure of spacetime, and the internal gauge theory comes as a breaking of the symmetry, triggered by $\boxed{\sigma}$, which now appears naturally has having to do with the geometry of spacetime.

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And this is all, for now. . .