

Pointers for a discussion on

Which geometry for universe at the big bang?

proposed, moderated, provoked and confused by

Fedele Lizzi

Università di Napoli *Federico II*

Institut de Ciències del Cosmo, Barcelona

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Plutarch: "Plato said God geometrizes continually"

So, which sort of geometry was he using when he decided to start the universe?

Let me start with a definition of Geometry

For which I can think of no higher authority than **Wikipedia**

Geometry (Greek γεωμετρία; geo = earth, metria = measure) is a part of mathematics concerned with questions of size, shape, and relative position of figures and with properties of space.

And this geometry has served us quite well for a few millennia

Or probably for more than a dozen billion years...

How reasonable is the assumption that spacetime at the time of the big bang was a “space”?

And if so, what sort of space was it?

Geometry existed before the creation

Plato

I think we all agree that the theory that we would have to use is quantum gravity. Which is a theory we do not have yet

Therefore an alternative question could be:

Which geometry for quantum gravity?

Failing the presence of a full universally accepted theory the question is in some sense idle

This is why this is a discussion session and not a seminar . . .

What we do have are attempts:

- String theory
- Loop Quantum Gravity
- Asymptotic safety, nonrenormalizable field theories Idots
- Discrete theories. Causal sets. Random lattices, graphs . . .
- Doubly Special relativity



please feel free to fill the dots

I have not put in the list *Noncommutative Geometry*.

The reason for this is that I think that NCG is a **tool**, not a theory

For example, quantum mechanics uses the noncommutative geometry of phase space in the Heisenberg formulation. But probably it is possible to view the Schrödinger formulation without considering operators and commutators. In my opinion you would not go very far, but it could be a point of view.

Strings

strings are a lot of things (too many?)

Coordinates of space-time are fields on a two-dimensional field theory

At least for the noninteracting case. Interaction is described by vertex operators, which form highly nontrivial algebras acting on the enormous Hilbert space of string states

One of the “problems” of string theory is the quest for the proper vacuum. But even if this is known, the geometry at very high energy, when the system would not be in equilibrium, and therefore far from the vacuum, is not at all clear.

And which role will play supersymmetry, which would be exact, for the geometry of spacetime?

Strings provide us with a very interesting phase transition: Hagedorn

That there is transition is rather clear. Its meaning in the hadronic case is also clear: **deconfinement**. But what it could mean in the spacetime case is much less clear!

Another important aspect certainly is T-duality. A doubling of spacetime?

In any case the historical great merit of string theory has been to make space “dynamical”.

Also loop quantum gravity is a lot of things. . .

This time spacetime takes the centre of the stage

Spin foams are what I would call a noncommutative geometry

Spacetime is described by a network of operator, it is a genuinely different idea of spacetime.

The problem on stage, as in many plays and movies, is the finale. Is not clear how to go from this quantum spacetime to the usual one.

If I may be provocative:

Loop quantum cosmology is like watching a sequel in a double bill, having fallen asleep and not really knowing how the first episode ended. . .

This is not to say that this kind of investigation is not important!
If we want to understand completely a theory before using it we would be discussing number theory

A good idea could be going to momentum space, and do away with spacetime which becomes a secondary concept, one can dispose of.

This is in a sense the idea of group field theory

But even in this case the classical limit is a problem

A discrete space?

Here by discretization I mean a fundamental property of spacetime, where the degrees of freedom are “finitary”, or locally countable in some sense, not discrete approximations.

A discrete graining of spacetime is fascinating. Also in this case there are some well defined phase transitions, and there is a better glimpse of what could be the other (“crumpled”) phase.

In the case of causal sets the fundamental structure of relativity is ingrained in the theory.

In these case the price we pay is the absence of symmetries.

It seems somehow strange to have a theory which puts together field theory and relativity doing without the fundamental symmetries

A possible alternative could be fuzzy space

The issue of symmetries may play a fundamental role

Quantum groups?

Here we go full circle and we get back to noncommutative geometry

Quantum groups re the symmetries of quantum space

Nonlocality: an asset or a problem?

But then, which quantum group?

κ -Poincaré is mathematically very interesting, in that it is a deformation of a fundamental symmetry of spacetime

It leads to κ -Minkowski, which is a space not exempt from problems in the definition of a field theory

Doubly special relativity? Relative locality? I will refrain from discussing it, hoping that someone else would do it...

Qual è 'l geomètra che tutto s'affige
per misurar lo cerchio, e non ritrova,
pensando, quel principio ond' elli indige,

tal era io a quella vista nova:
veder voleva come si convenne
l'imago al cerchio e come vi s'indova;

*As the geometrician, who endeavours
To square the circle, and discovers not,
By taking thought, the principle he wants,*

*Even such was I at that new apparition;
I wished to see how the image to the circle
Conformed itself, and how it there finds place;*

Dante, Paradiso, Canto XXXIII