Spectral data, spectral action,

renormalization, the Higgs and the Dilaton

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I will present a personal perspective of Connes noncommutative geometry construction of the standard model

It will be a physicist's overview , and I will be sketchy in the precise definition of the mathematical objects, and of the calculations of physical quantities. Thus treating equally bad both discipline, not to show preferences.

Calculations are often quite difficult and involved, and use mathematically ill defined quantities. Their value is in the fact that this kind of calculation often gives spectacularly accurate predictions which are verified with accuracy often of ten or more significant digits.

Any insertion of particle phenomenology into a larger framework immediately involves ramifications, and this may clash against experiments. Sometimes the models may fall immediately, other times they require the construction of whole laboratories to be killed, as in the case of the minimal $\boxed{SU(5)}$ grand unified theory. Labs were built to look for proton decay, which was not found. Fortunately the same labs found the mass of the neutrino.

Everything so far (and most of what I say later) can be found in the work of Connes, Lott, Chamseddine, Marcolli

There are some precursors for the role of the Higgs as coming from some sort of NCG in the work of Neeman, Madore, Kerner, Dubois-Violette.

A lot of work in this model has been dome by the Marseille group: Kastler, Iochum, Schucker, Krajewski, Martinetti, Stephan, and of course many more people that I have no time to mention

My own knowledge comes, apart from “reading the classics”, also from collaborations with Andrianov, Devastato, Figueroa, Gracia-Bondia, Kurkov, Mangano, Miele, Sparano, Varilly

You know the starting point of noncommutative geometry is the spectral triple

- A C^*-algebra $\mathcal{A}$ which describes the topology of spacetime.

- A Hilbert space $\mathcal{H}$ on which the algebra act as operators. This is always possible, and the representation need not be, and will in general not be, reducible.

- A (generalized) Dirac Operator D_0, self-adjoint and with compact resolvent. It contains the metric and differentiable properties.

There are two more ingredients to the “triple”:

- The chirality operator γ, with $\gamma^2 = \mathbf{I}$. This gives a $\mathbb{Z}_2$ grading and splits the $\mathcal{H} = \mathcal{H}_L \oplus \mathcal{H}_R$

- An antiunitary operator J, which gives the *real* structure to the noncommutative space. You may call it the Tomita-Takesaki operator, or charge conjugation.

In the case in which $\mathcal{A}$ is commutative this describes an ordinary topological space. And over the years a dictionary has been built to translate the usual geometrical concepts using these algebraic data.

For example bundles are projective modules, forms are built with the commutators between D_0 and $\mathcal{A}$ and are represented as operators on $\mathcal{H}$, the distance between points (pure states) can be built using the again the Dirac operator, etc.

The algebraic concepts are more robust than those based on "pointwise" geometry and they survive when the algebra is non-commutative, enabling us to do noncommutative geometry

In the commutative case it is possible to characterize a manifold with properties of the elements of the triple (all five of them)

There is a list of conditions and a theorem (Connes) which proves this.

Since the conditions are all purely algebraic there remain valid in the noncommutative case, defining a noncommutative manifold

In case you want to see them:

1. Dimension There is a nonnegative integer n such that the operator the Diximier trace of $|D_0|^{-1n}$ is finite.

2. Regularity For any $a \in \mathcal{A}$ both a and $[D_0, a]$ belong to the domain of δ^k for any integer k, where δ is the derivation given by $\delta(T) = [|D|, T]$.

3. Finiteness The space $\bigcap_k \text{Dom}(D^k)$ is a finitely generated projective left $\mathcal{A}$ module.

4. Reality There exist J with the commutation relation fixed by the number of dimensions with the property

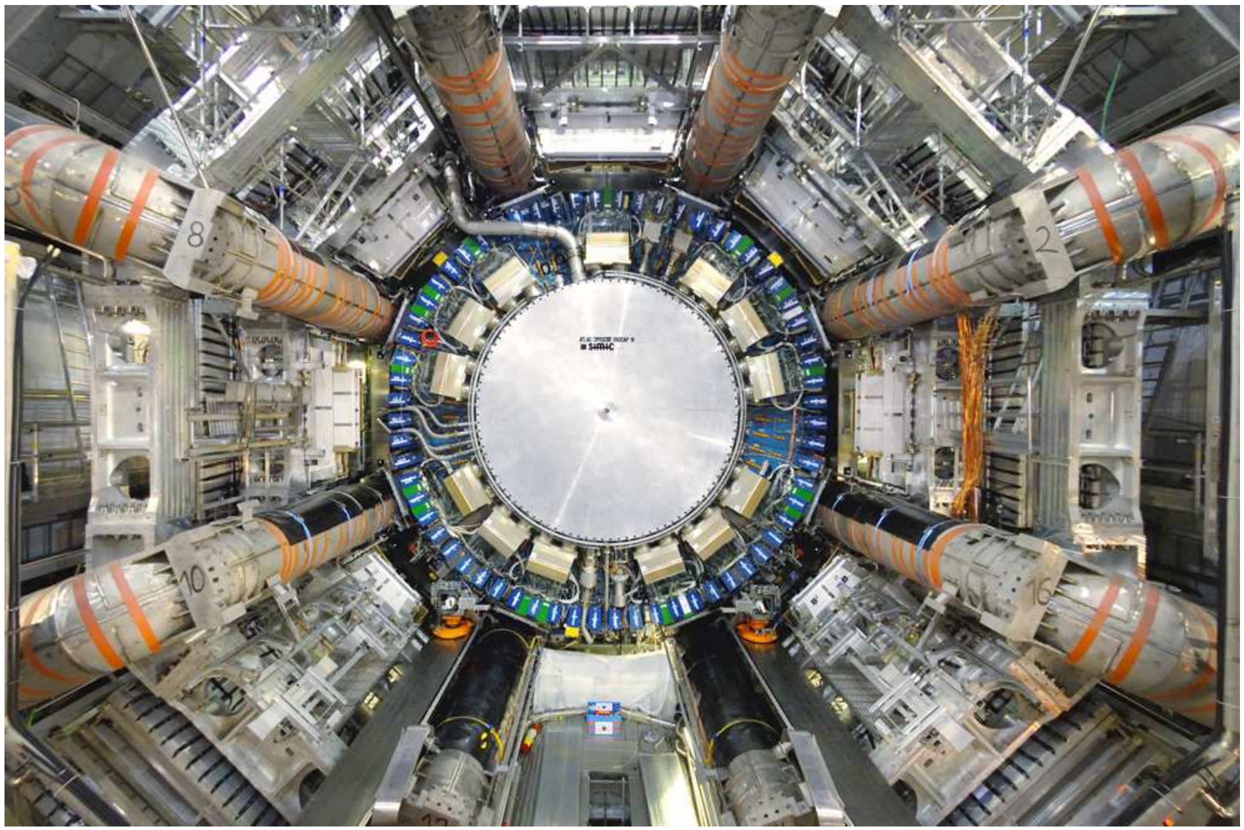
   (a) Commutant $[a, Jb^*J^{-1}] = 0, \forall a, b$

   (b) First order $[[D, a], b^o = Jb^*J^{-1}] = 0 \ , \forall a, b$

5. Orientation There exists a Hochschild cycle c of degree n which gives the grading γ , This condition gives an abstract volume form.

6. Poincaré duality A Certain intersection form detemrined by D_0 and by the K-theory of $\mathcal{A}$ and its opposite is nondegenrate.

So what has this to do with the **Large Hadron Collider** ?



A particularly simple form of noncommutative geometry describes the standard model of particle interaction, the model investigated at CERN

The noncommutative geometry is particularly simple because it is the product of an infinite dimensional commutative algebra times a noncommutative finite dimensional one

Hence this algebra, being Morita equivalent (i.e. having the same representations) of the commutative one describes a mild generalization of the space

The infinite dimensional part is the one relative to the four dimensional spacetime

It is worth mentioning that everything works only if this spacetime is compact and Euclidean, which is not the case in the "real" world. But in this case we are in good company, often in physics field theories are built on these "bad" spaces

We start from the algebra, a tensor product $\boxed{\mathcal{A} = C(\mathbf{R}^4) \otimes \mathcal{A}_F}$, with the finite $\boxed{\mathcal{A}_F = \text{Mat}(\mathbb{C})_3 \oplus \mathbb{H} \oplus \mathbb{C}}$

The unitaries of the algebra correspond to the symmetries of the standard model: $\boxed{SU(3) \oplus SU(2) \oplus U(1)}$

A unimodularity condition takes care of the extra U(1)

This algebra must be represented as operators on a Hilbert space, which also has a continuos infinte dimesional part (spinors on spacetime) times a finite dimensional one: $\boxed{\mathcal{H} = \text{sp}(\mathbf{R}) \otimes \mathcal{H}_F}$. The grading given by $\boxed{\gamma}$ splits it into a left and right subspace: $\boxed{\mathcal{H}_L \oplus \mathcal{H}_R}$

The $\boxed{J}$ operator basically exchange the two chiralities and conjugates, thus effectively making the algebra act form the right.

For $\boxed{\mathcal{H}_\mathcal{F}}$ we take the zoo of known fermions:

- Quarks, which come in three colours, i.e. transform under the fundamental representation of colour $\boxed{SU(3)}$

- Leptons, which are singlets (trivial representation) under $\boxed{SU(3)}$

All fermions come in a left and a right "chirality", belonging to $\boxed{\mathcal{H}_{L,R}}$, they transform respectively under the fundamental or trivial representation of $\boxed{SU(2)}$

The representations under $\boxed{U(1)}$ are different, there are different hypercharges, but their value is arranged in a such a way to "miraculously" cancel potentially dangerous *anomalies* which could occur in the gauge field theory.

An then you take the whole package and replicate it in three identical versions (generations), differing only for the mass of the particles.

In total there are 96 degrees of freedom, including right handed neutrinos, a relatively recent acquisition in the zoo.

Now one has to take this complicated Hilbert space, and try to find a representation of $\boxed{\mathcal{A}_F}$ in such a way that the fermions transform properly under the relevant groups, and the conditions involving $\boxed{\mathcal{A}, \gamma}$ and $\boxed{J}$ are satisfied

Then one has to define a significant $\boxed{D_0}$ which will satisfy the remaining conditions, and which will have physical relevance (more later).

The good news is that this is possible

Moreover it turns out that the stringent conditions basically constrain the algebra, and hence the symmetries of the standard model, to be the one above

The use of $\boxed{C^*}$-algebras, instead of groups, severely restricts the representations. Very few gauge theories can be described by a noncommutative manifolds

Fortunately the standard model can.

I still have to tell you what D_0 operator is. It will carry the metric information on the continuous part as well as the internal part.

For this almost commutative geometry it will be again split into a continuous and a finite part

$$D_0 = \gamma^\mu(\partial_\mu + \omega_\mu) \otimes \mathbf{I} + \gamma^5 \otimes D_F$$

ω_μ the spin connection. The presence of γ^5, the chirality operator for the continuous manifold is for technical reasons.

All of the properties of the internal part are encoded in D_F, which is a 96×96 matrix.

The Dirac operator appears in the fermionic part of the action, which describes the motion of fermions as

$$\langle\Psi|\, D_0 \,|\Psi\rangle$$

The part of D_0 containing the spin connection, i.e. the curvature of spacetime gives the coupling with the gravitational field

The finite part must provide the couplings between the left and right components of the particles, the mass terms, for the electron

$$\langle e_R|\, m_e \,|e_L\rangle$$

In D_0 we have allowed for a non trivial connection in the manifold, but with can allow for a nontrivial connection also in the inner, gauge sector

$$D = D_0 + A + JAJ$$

where A is a connection one form, which in this context is an operator of the form $\sum_i a_i[D_0, b_i]$, with $a_i, b_i \in \mathcal{A}$

We can then write the fermionic action

$$S_F = \langle\psi| D |\psi\rangle$$

I will not write the explicit form of the Dirac operator, but I want to stress that it is already a nontrivial feat to be able to write the complicated form of the standard model action is such a way

This action represents the classical motion of fermionic field in a fixed background given by the nontrivial connection

This is already a nontrivial feat, it is remarkable that the standard model can be cast in this framework, but not all gauge theories can

Still, from a physicist's point of view, this has nothing to do with LHC, we have to quantize the theory

And quantization of infinite dimensional things is never an easy task

We start from the partition function, which is the starting point for the path integral quantization

$$Z(D) = \int [\mathrm{d}\Psi][\mathrm{d}\bar{\Psi}] e^{-S_F}$$

This is an integral over (fermionic) Grassman variables, and it looks formally like a determinant, but to write it we need a normalization scale, with the dimensions of an energy

I use units of measurement given by the fundamental constants $\hbar, c, G$ which I set to be the of magnitude 1

We need a scale to regularize the theory. The expression of the partition function can be formally written as a determinant, introducing a normalization dimensional constant $\boxed{\mu}$:

$$\boxed{Z(D,\mu) = \int [\mathrm{d}\psi][\mathrm{d}\bar{\psi}] e^{-S_F} = \det\left(\frac{D}{\mu}\right)}$$

The determinant is still infinite, since the system is infinite dimensional.

Therefore we need a procedure which first will regularize the theory. Since the problem is in the the fact that the eigenvalues of $\boxed{D}$ grow we need to stop them growing. This means that we have to put a cutoff on them, which means a cutoff on the energies

then we will have to decide what to do with this cutoff

The regularization can be done in several ways. In the spirit of noncommutative geometry the most natural one is a truncation of the spectrum of the Dirac operator. This was considered long ago by Andrianov, Bonora, Fujikawa, Novozhilov, Vassilevich

The cutoff is enforced considering only the first N eigenvalues of D

Consider the projector $P_N = \sum_{n=0}^{N} |\lambda_n\rangle \langle\lambda_n|$ with λ_n and $|\lambda_n\rangle$ the eigenvalues and eigenvectors of D

N is a function of the cutoff defined as $N = \max n \text{ such that } \lambda_n \leq \Lambda$

We effectively use the N^{th} eigenvalue as cutoff

The choice of a sharp cutoff could be changed in favour of a smooth cutoff function χ which weights the eigenvalues less and less as they grow

Define the regularized partition function

$$Z(D,\mu) = \prod_{n=1}^{N} \frac{\lambda_n}{\mu} = \det\left(\mathbb{1} - P_N + P_N \frac{D}{\mu} P_N\right)$$

All physical quantities which one can calculate, like masses, or couplings (strength of the interaction) will be function of these normalizations and cutoff, which have no physical meaning

This is solved by the renormalization group programme. In a nutshell this governs the change of physical constants under a change of the energy scale of normalization

This means that their derivatives of certain functions under a change of the scale vanishes

This implies that masses, (and couplings) will not have a fixed values, but will run with energy. All predictions of a quantum field theory are quantities which are valid at a certain energy.

Under the change $\boxed{\mu \to \gamma\mu}$ the partition function changes

$$\boxed{Z(D,\mu) \to Z(D,\mu)e^{-\log\gamma\,\mathrm{tr}\,P_N}}$$

On the other side

$$\boxed{\mathrm{tr}\,P_N = N = \mathrm{tr}\,\chi\left(\frac{D}{\Lambda}\right) = S_B(\Lambda, D)}$$

for the choice of $\boxed{\chi}$ the characteristic function on the interval, a consequence of our sharp cutoff on the eigenvalues.

We found the spectral action.

This has been introduced by Chamseddine and Connes in 1996, as a starting point. Here we see that it actually emerges naturally form the NCG spectral point of view.

The spectral action gives dynamics to all background fields, the the gauge bosons, which are present if the internal part of the connection, and couple them with gravity, present in the Levi-Civita connection.

The calculations for the spectral action can be done with heath kernel techniques. After all it is just cranking a machine...

But the cranking depends on the machine

$$\begin{gathered}
\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \\
\frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - \\
W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - igs_w(\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - \\
W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + \\
g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w (A_\mu Z^0_\nu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \frac{1}{2}\partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
\beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \frac{2M^4}{g^2}\alpha_h - g\alpha_h M \left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\frac{1}{8}g^2\alpha_h \left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - gMW^+_\mu W^-_\mu H - \\
\frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\frac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\left(\frac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - \\
ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2W^+_\mu W^-_\mu\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \\
\frac{1}{8}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-\right) - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \\
\frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - \\
g^2\frac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - g^2s_w^2A_\mu A_\mu\phi^+\phi^- + \frac{1}{2}ig_s\,\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + \\
m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_wA_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\frac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + (\bar{u}^\lambda_j\gamma^\mu(1 - \frac{8}{3}s_w^2 + \\
\gamma^5)u^\lambda_j)\} + \frac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5){U^{lep}}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\frac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa {U^{lep}}^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda {U^{lep}}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda {U^{lep}}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda {U^{lep}}^\dagger_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda {U^{lep}}^\dagger_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \frac{g}{2}\frac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \frac{g}{2}\frac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) + \\
\frac{ig}{2}\frac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \frac{ig}{2}\frac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5e^\lambda) - \frac{1}{4}\,\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa - \frac{1}{4}\,\overline{\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa} + \\
\frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_jC_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_jC_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \frac{g}{2}\frac{m^\lambda_u}{M}H(\bar{u}^\lambda_ju^\lambda_j) - \frac{g}{2}\frac{m^\lambda_d}{M}H(\bar{d}^\lambda_jd^\lambda_j) + \\
\frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5d^\lambda_j)
\end{gathered}$$

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Here the notation is as in [46], as follows.

And now we get close to LHC

The remarkable fact is that the fluctuations of the Dirac operator introduced the bosonic fields, gluons which are responsible for the strong (nuclear) force, the $\boxed{W}$ and $\boxed{Z}$ bosons responsible for the weak force, the photon and another field, which in view of its coupling to the fermion is responsible for the breaking of the symmetry and to give mass to the fermions.

This is the Higgs (Englert, Brout, Guralnick, Hagen, Kibble) boson.

There is of course reason to be satisfied, but is this all?

We should get numbers. In a form which can be confronted with experiment. And while we cannot aim at agreement with 12 significant digits, at least two or three...

It is already nice to be able to explain with a single model inserted in a larger framework the existing data (for example elliptic and gravitational law orbits vs. epicycloids and a geocentric model)

But it is important to have some predictive power. Finding Neptune in the right spot for example.

We do have a Neptune on sight, actually the Neptune. It turns out that the parameters of the Higgs, including its mass, are a function of the masses of the other particles!

But we are not yet there...

The impressive lagrangian written above is still classical, one has to quantize it and implement on it the renormalization programme

Renormalization means that all constant quantities in the action are a functions of the energy: running coupling constants

The running is given by the solution of an ordinary differential equation, called the $\boxed{\beta}$ function, calculated perturbatively to first (occasionally second, rarely third) order in $\boxed{\hbar}$ by the $\boxed{n}$-point amplitude (loop expansion)

I wish to stress that I am absolutely not an expert in this kind of calculations, but if one wants to play this game one has to do it...

The three-point vertex insertion diagrams of class (ii) will similarly be denoted by (m/V), where m is the diagram of fig. 1 in which the insertion is made. The singular terms associated with these diagrams are given in table 2. The group theoretic factors introduced in these diagrams in addition to those in eqs. (3.3) – (3.10) are defined by

$$\overline{H}^{Y}_{abcd} \equiv \sum_{\text{perms}} \mathrm{Tr}\left(Y^{e}Y^{\dagger a}Y^{e}Y^{\dagger b}Y^{c}Y^{\dagger d}\right), \tag{3.13}$$

$$H^{S}_{abcd} \equiv \sum_{k} C_2(k) H_{abcd}. \tag{3.14}$$

The four-point vertex insertion diagrams and the single pole diagrams, which will be analyzed together, are most conveniently classified into groups containing the same power of the gauge coupling constant.

The diagrams independent of the gauge coupling are shown in fig. 2, and the associated singular terms are given in table 3. The group theoretic factors associated with these diagrams are defined by

$$\Lambda^{3}_{abcd} \equiv \tfrac{1}{8} \sum_{\text{perms}} \lambda_{abef}\lambda_{efgh}\lambda_{ghcd}, \tag{3.15}$$

$$\overline{\Lambda}^{3}_{abcd} \equiv \tfrac{1}{4} \sum_{\text{perms}} \lambda_{abef}\lambda_{cegh}\lambda_{dfgh}, \tag{3.16}$$

$$H^{\lambda}_{abcd} \equiv \tfrac{1}{2} \sum_{\text{perms}} \lambda_{abef} \mathrm{Tr}\left(Y^{c}Y^{\dagger d}Y^{e}Y^{\dagger f}\right), \tag{3.17}$$

$$\overline{H}^{\lambda}_{abcd} \equiv \tfrac{1}{4} \sum_{\text{perms}} \lambda_{abef} \mathrm{Tr}\left(Y^{c}Y^{\dagger e}Y^{d}Y^{\dagger f}\right), \tag{3.18}$$

$$H^{3}_{abcd} \equiv \tfrac{1}{2} \sum_{\text{perms}} \mathrm{Tr}\left(Y^{a}Y^{\dagger b}Y^{e}Y^{\dagger c}Y^{d}Y^{\dagger e}\right). \tag{3.19}$$

TABLE 3

Singular parts of the scalar quartic vertex renormalization Z_{abcd}, as defined by eq. (3.1), for the diagrams shown in fig. 2 which are independent of the gauge coupling constant

Diagram	S_{abcd}	A	B
(2.1)	Λ^{3}_{abcd}	$-\frac{1}{4}$	0
(2.2)	$\overline{\Lambda}^{3}_{abcd}$	$-\frac{1}{4}$	$\frac{1}{4}$
(2.3)	$\kappa H^{\lambda}_{abcd}$	2	0
(2.4)	$\kappa \overline{H}^{\lambda}_{abcd}$	2	-2
(SP1)	κH^{3}_{abcd}	0	-2

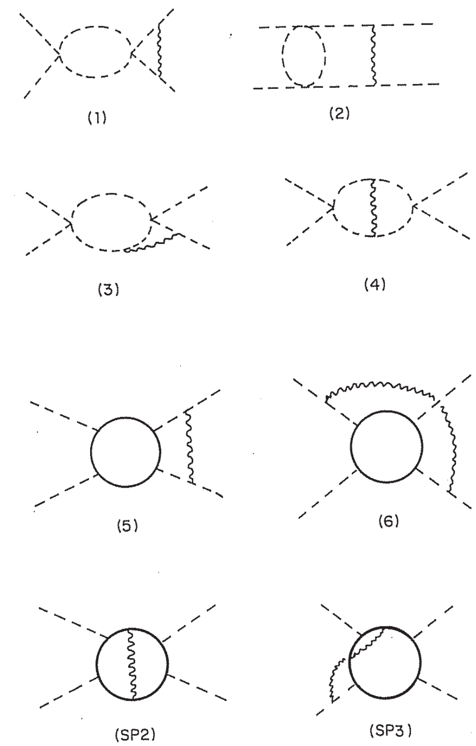


Fig. 3. Two-loop corrections to the proper scalar quartic vertex of order g^2.

TABLE 4

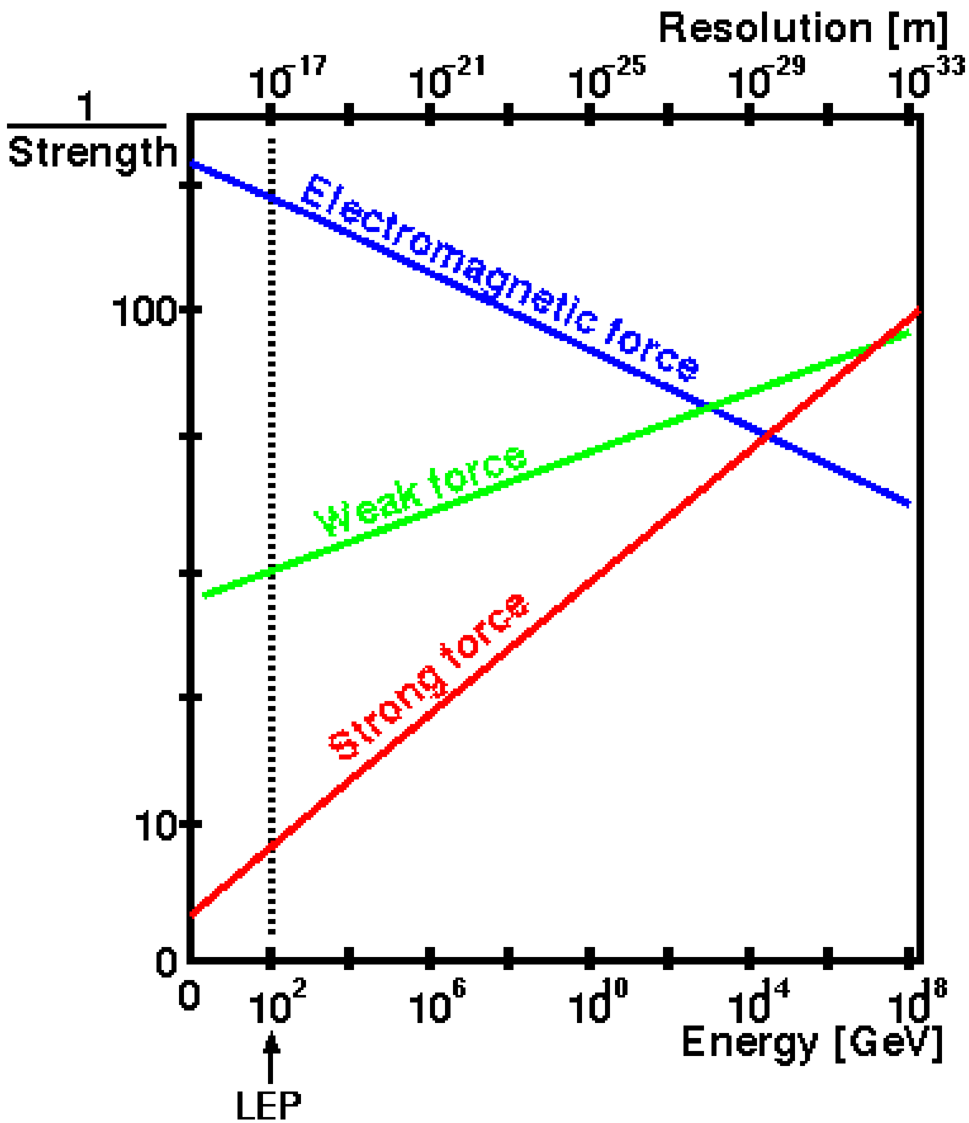
Singular parts of the scalar quartic vertex renormalization Z_{abcd}, as defined by eq. (3.1), for the diagrams of order g^2 shown in fig. 3

Diagram	S_{abcd}	A	B
(3.1)	$\overline{\Lambda}^{2S}_{abcd} + \Lambda^{2g}_{abcd} - \frac{3}{4}\Lambda^{2S}_{abcd}$	$-(1-\alpha)$	0
(3.2)	$\overline{\Lambda}^{2S}_{abcd} + \Lambda^{2g}_{abcd}$	$\frac{1}{2}(1-\alpha)$	$-\frac{1}{2}(1-\alpha)$
(3.3)	$\overline{\Lambda}^{2S}_{abcd} + \Lambda^{2g}_{abcd}$	$1-\alpha$	$1-\alpha$
(3.4)	Λ^{2g}_{abcd}	$-\frac{1}{2}(1-\alpha)$	$\frac{1}{2}(2+\alpha)$
(3.5)	$\kappa(H^{g}_{abcd} + H^{S}_{abcd} - \frac{1}{2}H^{F}_{abcd})$	$-2(1-\alpha)$	0
(3.6)	$\kappa(H^{g}_{abcd} + \frac{1}{2}H^{S}_{abcd} - \frac{1}{2}H^{F}_{abcd})$	$2(1-\alpha)$	$-2(1-\alpha)$
(SP2)	κH^{g}_{abcd}	0	$-2(1-\alpha)$
(SP3)	$\kappa(2H^{g}_{abcd} + H^{S}_{abcd} - H^{F}_{abcd})$	0	$2(1-\alpha)$

The first think one has to decide is at which energy one write the big lagrangian. This will give a boundary condition

One boundary condition can be given by the fact that in the model obtained cranking the machine the strength of the fundamental interaction is equal

Experimentally is known that *if there are no other particles appearing at higher energy* the three coupling constant are almost equal in one point:



Resolution [m]
10^{-17}
10^{-21}
10^{-25}
10^{-29}
10^{-33}
1
Strength
100
10
0
Electromagnetic force
Weak force
Strong force
0
10^{2}
10^{6}
10^{10}
10^{14}
10^{18}
Energy [GeV]
LEP

In the original Chamseddine Connes Marcolli the unification point was to taken the one in which the two nonabelian force come together. Then the calculation was performed using several articles using different normalizations, correcting each other for the various coefficients, using Mathematica to solve the equation . . .

The number quoted for the mass of the Higgs boson is 170 GeV

Which is definitively accurate to one digit

This is nontrivial, we started with a purely geometrical framework, with parameters spanning a range of four order of magnitude (many more if one counts neutrino masses), so that the mass of the Higgs could have come to be a fraction of the electron mass!

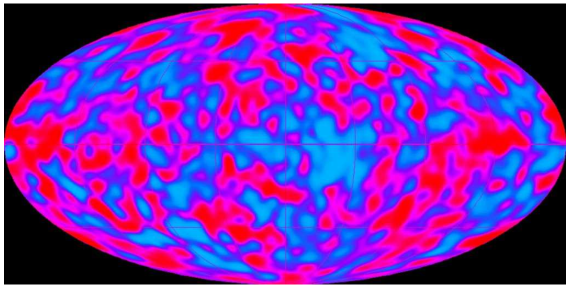
The mass of the Higgs is presently not yet known, the particular 170 GeV value has been excluded, and there is some experimental signal at 126 GeV

We have to look for some other aspects of the theory if we want to improve on the model

While at the same time trying not to spoil the mathematical beauty of the model, as well as its insertion in general framework

One ingredient: anomalies: classical symmetries which are not respected by quantization

Another ingredient: input from the biggest experiment: The big Bang and its evolution.



The following is in collaboration with Andrianov and Kurkov JHEP 1005:057,2010; JHEP 1110:2011:001

The classical action is invariant for the following transformation

$$|\Psi\rangle \to \mathrm{e}^{\frac{1}{2}\phi}|\Psi\rangle$$

$$D \to \mathrm{e}^{-\frac{1}{2}\phi} D \mathrm{e}^{-\frac{1}{2}\phi}$$

Recalling the presence of $\sqrt{\det g}$ in the integral for the position representation of the Hilbert space it is easy to see that this is actually related to Weyl rescaling

$$g^{\mu\nu} \to \mathrm{e}^{2\phi} g^{\mu\nu}$$

This is a symmetry of the classical action, not of the regularized quantum partition function

$$Z(D) = \int [\mathrm{d}\psi][\mathrm{d}\bar{\psi}] e^{-S_\psi}$$

therefore there is an anomaly because a classical Weyl symmetry is not preserved at the quantum level by a regularized diffeomorphism invariant measure.

One can now perform some standards calculations (which I spare) splitting the partition function in the product of a term invariant for Weyl transformations, and another not invariant, which will depend on the field $\boxed{\phi}$, the dilaton.

The dilaton becomes a collective mode of the fermions, mediating the breaking of Weyl symmetry

We assume therefore the presence, in an earlier epoch, of a conformal point, in which the symmetry is restored. A phase in which all particles are massless, and the Higgs potential does not have the degenerate minimum

We can calculate now the noninvariant part of the bosonic action

$$S_{not} = \ln \frac{Z(D_\phi}{Z(D)}$$

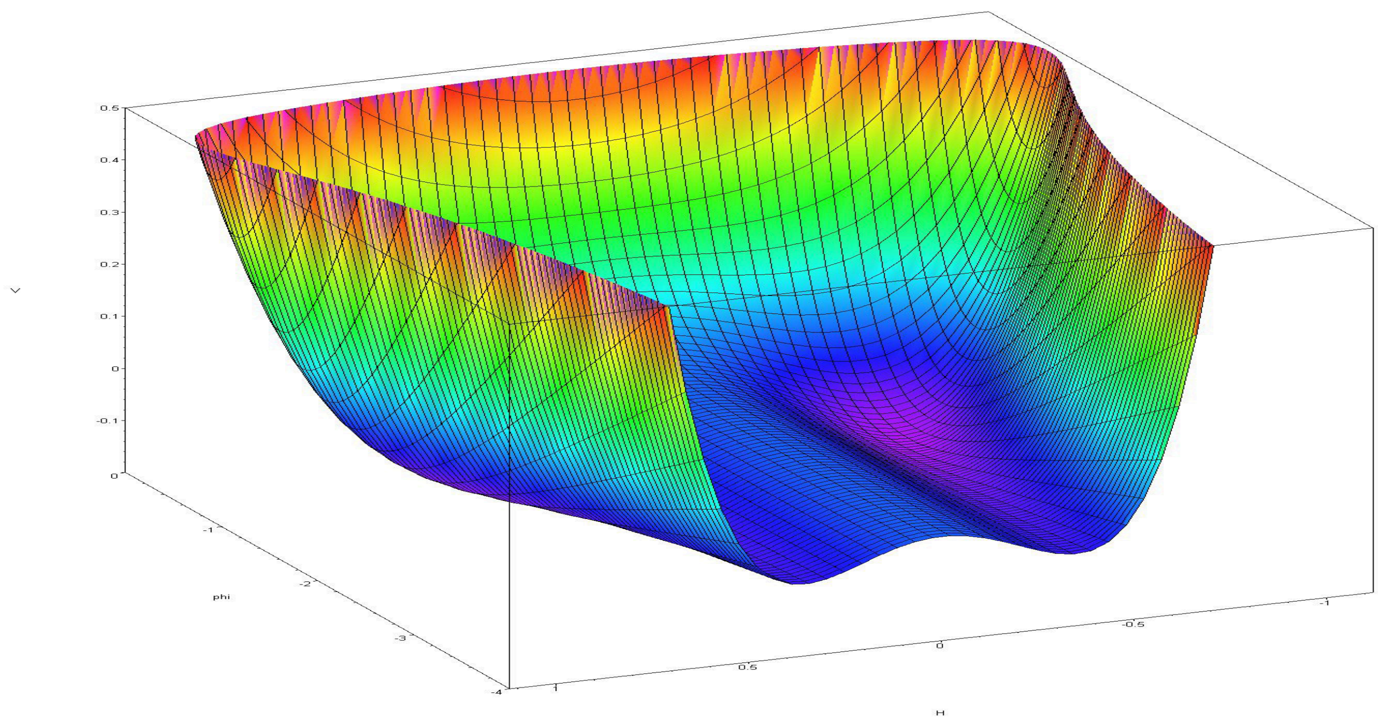
The normalization constant μ can be fixed requiring that the constant term in the action proportional to Λ^4 vanishes. This gives

$$\Lambda = e^{\frac{1}{4}}\mu$$

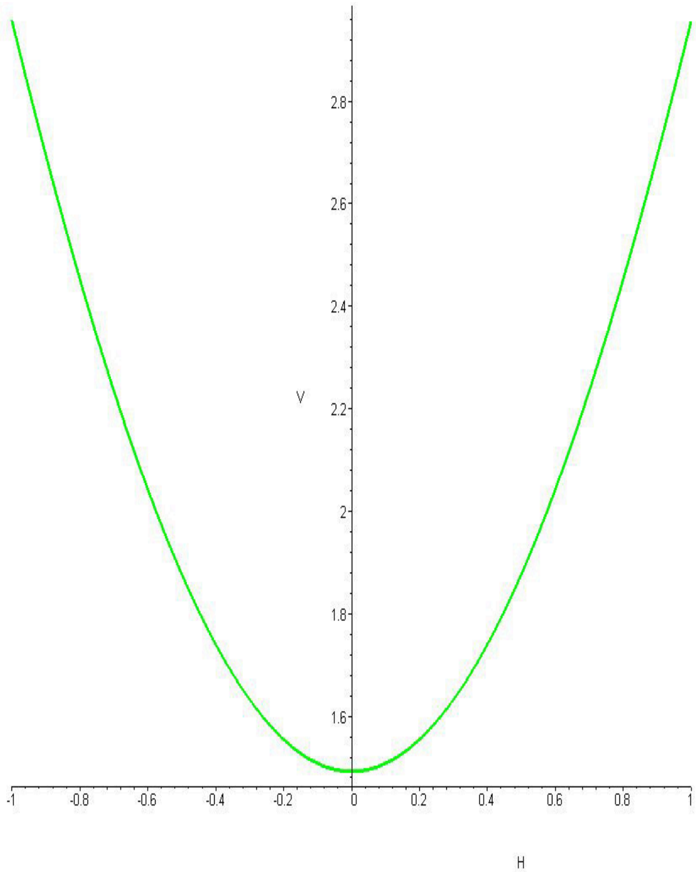
It is then possible to evolve the potential and find the following properties for the Higgs-dilaton potential

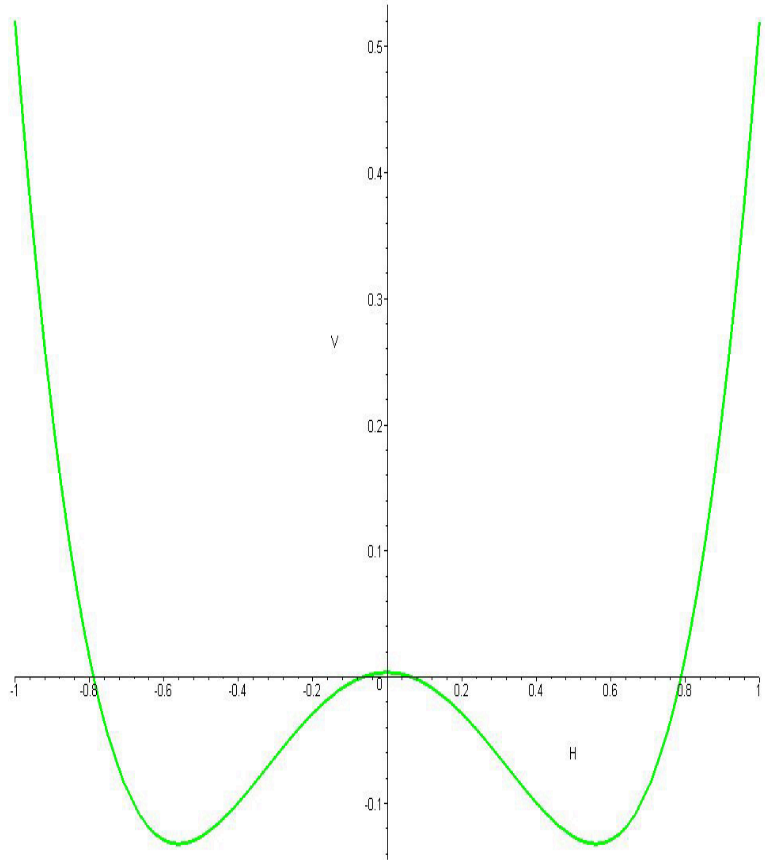
- The existence of a local minimum
- The existence of an unbroken phase from which the potential may roll

Plot of the effective Higgs-Dilaton potential:



We see that for different values of $\boxed{\phi}$, the potential $\boxed{V(H)}$ has a transition from a symmetric to a broken phase.





What I have describe so far referred to the case of a constant dilaton.

It is possible to actually calculate explicitly the action of the collective modes:

$$S_{coll} = \Lambda^2 \left(\alpha_1 (H^2 - \alpha_2 R)(e^{2\phi(x)} - 1) + \alpha_3 e^{2\phi(x)} \phi_{;\mu} \phi_{;}{}^{\mu} \right)$$

$$+ \alpha_4 \left(\phi_{;\mu}{}^{\mu} + \phi_{;\mu} \phi_{;}{}^{\mu} \right) \left(\alpha_5 H^2 - \frac{1}{3} \cdot R - \phi_{;\mu}{}^{\mu} - \phi_{;\mu} \phi_{;}{}^{\mu} \right) \Big)$$

$$-\alpha_6 \phi(x) \left(H_{;\mu} H_{;}^{\mu} - \frac{1}{6} R H^2 + \alpha_7 H^4 + \alpha_8 C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} \right.$$

$$\left. + \alpha_9 F^{\mu\nu} F_{\mu\nu} + \alpha_{10} W_{\mu\nu} W^{\mu\nu} + \alpha_{11} G_{\mu\nu} G^{\mu\nu} \right) + \text{invariant part}$$

where the α_i are positive numbers, which depend on the couplings

The qualitative behaviour does not change, we are in the process of "putting the numbers in"

Conclusions

- What remains to be seen is if from the particle physics point of view the noncommutative geometry is Kepler's law, the theory of gravitation cum differential calculus, the law of diminishing proportions of Hooke, some further epicycloid or Kant's theory of heavens.

- Unfortunately we have to get our hand quite dirty in the process

- But then, also the author of the paintings in this room did have the same dirty hand of Michelangelo and the mason who built the room