# Quantum Field Theory:

# A Spectral Point of View

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# What is Quantum Field Theory?

The presence of a question mark in the title of the workshop puts an heavy responsibility on the speakers. I feel I have to give an answer to the question.

Moreover, I teach the Quantum Field Theory class at my University. So lack of an answer not only may jeopardize my right to have dinner here, but my job as well!

On the other side there is the fact that notoriously all I can do is to multiply matrices, better if they are $\boxed{2 \times 2}$.

Therefore I will try to give an answer using the least sophisticated tools I can find. I will be schematic to the extreme and sacrifice not only rigour, but also precision and even correctness for the sake of giving an impressionistic but complete view.

To do a Quantum Field theory the main ingredient is: *Fields*

They are vectors on an Hilbert space which transform under some representation of some groups which I call symmetries.

One of these symmetries is the Lorentz group, and fields are either fermions or bosons. Matter fields in nature are almost always fermions, while bosons appear as the mediators of the forces related to the symmetries, in other words they appear in the covariant derivative.

The almost above is there because of the Higgs boson

An important ingredient is the chirality operator, which I will call $\boxed{\gamma}$, with $\boxed{\gamma^2 = 1}$, so that the Hilbert space of fermions splits into the two eigenspaces of the chirality:

$$\boxed{\mathcal{H} = \mathcal{H}_L \oplus \mathcal{H}_R}$$

To have dynamics of the fermions I need a propagator, coming form an action. And I will schematically write

$$\boxed{S_F = \langle\Psi|\, D\, |\Psi\rangle}$$

with $\boxed{D}$, the inverse of the propagator, is an operator which I will call generically the Dirac operator.

$\boxed{D}$ is a self-adjoint, unbounded operator and as such it can be diagonalized. Since I declared that I only know matrices I will assume that is has discrete spectrum. This means that I am considering Euclidean and compact theory. A sin that I hope will be forgiven. . .

But actually, how do I know that I am *somewhere*, so far I have only introduced vectors and operators (matrices). One operator, the Dirac operator plays a dynamical role, but what happened to spacetime?

In the spectral spirit of this talk I will consider a set of operators which form a commutative algebra and call this spacetime

Here I implicitly using the machinery of noncommutative geometry, which is of course the inspiration behind all this.

Any commutative $\boxed{C^*}$-algebra is always the algebra of continuous functions of topological space. Therefore the identification of a commutative subalgebra $\boxed{\mathcal{A}}$ of the observables of my Hilbert space gives me the topology of space.

The Dirac operator then encodes the geometry. It gives the distance among the points of space, hence the metric. The growth of its eigenvalues gives the (spectral) distance. It gives a representation of the exterior forms algebra...

These three main ingredients, $\boxed{\mathcal{H}, D, \mathcal{A}}$ form the basis of Connes' spectral triple. Essential seasonings are the chirality $\boxed{\gamma}$ which I already introduced, and the charge conjugation operator $\boxed{J}$. An antilinear operator.

We have the action, which so far is the action of fermions in a fixed background, because I am not yet letting $\boxed{D}$ fluctuate. We can therefore proceed to quantize. Write down the partition function

$$\boxed{Z(D) = \int [\mathrm{d}\psi][\mathrm{d}\bar{\psi}] e^{-S_F}}$$

This expression is formally the determinant of $\boxed{D}$, but to actually give it a meaning I need to do some work.

First of all I can only calculate determinants of dimensionless quantities, therefore I need a scale $\boxed{\mu}$ to divide $\boxed{D}$ and write (still formally)

$$\boxed{Z(D) = \det\left(\frac{D}{\mu}\right)}$$

The expression is still formal because the matrix is infinite, and he eigenvalues of $\boxed{D}$ are growing

Therefore I need to regularize the theory

In the spirit of this talk most natural regularization is a truncation of the spectrum of the Dirac operator at some scale $\boxed{\Lambda}$.

The cutoff is enforced considering only the first $\boxed{N}$ eigenvalues of $\boxed{D}$

Consider the basis of eigenfunctions of $\boxed{D}$ ordered in increasing values of the eigenvalues (with repetitions due to degeneracy):

$$\boxed{D\,|\lambda_n\rangle = \lambda_n\,|\lambda_n\rangle}$$

the integer $\boxed{N}$ defined as

$$\boxed{N = \max n \text{ such that } \lambda_n \leq \Lambda}$$

and the projector

$$\boxed{P_N = \sum_{n=0}^{N} |\lambda_n\rangle\,\langle\lambda_n|}$$

We effectively use the $\boxed{N^{\text{th}}}$ eigenvalue as cutoff

Define the regularized partition function

$$\boxed{Z(D,\mu) = \prod_{n=1}^{N} \frac{\lambda_n}{\mu} = \det\left(\mathbb{1} - P_N + P_N \frac{D}{\mu} P_N\right)}$$

$$\boxed{= \det\left(\mathbb{1} - P_N + P_N \frac{D}{\Lambda} P_N\right) \det\left(\mathbb{1} - P_N + \frac{\Lambda}{\mu} P_N\right)}$$

$$\boxed{= Z_\Lambda(D,\Lambda) \det\left(\mathbb{1} - P_N + \frac{\Lambda}{\mu} P_N\right)}$$

The cutoff $\boxed{\Lambda}$ can be given the physical meaning of the energy in which the effective theory has a phase transition, or at any rate an energy in which the symmetries of the theory are fundamentally different (unification scale)

The quantity $\boxed{\mu}$ in conceptually different from $\boxed{\Lambda}$ and is a normalization scale, which changes with the renormalization flow

Under the change $\boxed{\mu \to \gamma\mu}$ the partition function changes

$$\boxed{Z(D,\mu) \to Z(D,\mu)e^{-(\log\gamma)\,\mathrm{tr}\,P_N}}$$

On the other side

$$\boxed{\mathrm{tr}\,P_N = N = \mathrm{tr}\,\chi\left(\frac{D}{\Lambda}\right)}$$

where $\boxed{\chi}$ is the characteristic function on the interval. Had we chosen smooth cutoff of the eigenvalues this function would reflect this choice.

The renormalization flow forces us to add another term to the action. This is the Spectral Action introduced by Chamseddine and Connes

$$\boxed{S_B(\Lambda, D) = \mathrm{tr}\,\chi\left(\frac{D}{\Lambda}\right)\ \mathrm{tr}\,\chi\left(\frac{D^2}{\Lambda^2}\right)}$$

It is again a spectral object. A regularized trace

Now we are considering $\boxed{D}$ as a dynamical object, and the dynamics are consequence of its fluctuations

In particular consider it as the sum of a background part, and of a connection part

$$\boxed{D = D_0 + A}$$

So far I have totally generic. Let me consider some examples

The calibrating example is to consider as Hilbert space of the usual fermions. As algebra that of continuous functions on spacetime, and as Dirac operator what we usually call the Dirac operator, which we consider to be the sum of a backgroung operator, plus a fluctuating part, the connection

$$\boxed{D = D_0 + \gamma^\mu A_\mu}$$

$$\boxed{D = \gamma^\mu \partial_\mu + e\gamma^\mu A_\mu}$$

From these data it is possible to reconstruct, in a purely algebraic way, the geometry of spacetime, its metric, cohomology, bundles etc. This is from a mathematical point of view the essence of Connes noncommutative geometry programme.

The fermionic action is the usual one, so no surprises there. As for the bosonic part, it can be calculated using heath kernel techniques.

For any pseudo differential operator $\boxed{A}$ of order $\boxed{m}$ in $\boxed{d}$ dimensions

$$\boxed{\mathrm{Tr}\,(A^{-s}) = \frac{1}{\Gamma(s)} \int_0^\infty \mathrm{d}t\,(t(s-1)\,\mathrm{Tr}\,\mathrm{e}^{-tA})}$$

and expanding

$$\boxed{\mathrm{Tr}\,\mathrm{e}^{-tA} = \sum_n t^{\frac{n-d}{m}} \int_M \sqrt{g}\mathrm{d}^d x a_n(x,A)}$$

Considering a smoothened version of $\boxed{\chi}$, which must be analytic, the expansion becomes a sum of residues

$$\boxed{\mathrm{Tr}\,\chi(A) = \sum_n f_{2n}(\chi) a_{2n}(A)}$$

with $\boxed{f_0 = \int \mathrm{d}x\, x\,\chi(x);\;\; f_2 = \int \mathrm{d}x\,\chi(x);\;\; f_{2n+2} = (-)^n \chi^{(n)}(0)}$

The heath kernel was developed for operators of the Laplacian type, which luckily is our case, therefore we can just read the coefficients (Seeley-De Witt) $\boxed{a_n}$ from a book

The coefficient $\boxed{a_0}$ is always constant and proportional to the volume of spacetime times $\boxed{\Lambda^4}$

In this particular case $\boxed{a_2 = 0}$ and

$$\boxed{a_4 = -\frac{1}{4}\frac{1}{12}F^{\mu\nu}F_{\mu\nu}}$$

and the curious $\boxed{\frac{1}{12}}$ is the value of the fine structure constant at the "unification scale" $\boxed{\Lambda}$

Apart from this fine tuning we have found electrodynamics

And now let us go for the big price: The standard model

In order to this I will have to generalize the commutative algebra, which in the commutative case describe an ordinary manifold, to a noncommutative algebra describing a noncommutative space

Actually the generalization will be quite mild, to the product of a continuous commutative geometry times a finite noncommutative internal space

First another bit of spectral mathematics. The game is to represent all of geometry form a spectral point of view. I have given you the spectral interpretation for a topological metric space, the spectral triple. But which condition should be imposed in order to describe a manifold?

There are seven conditions that the package $\boxed{\mathcal{A}, \mathcal{H}, D, \gamma, J}$ must satisfy so that the resulting space is a manifold for a commutative $\boxed{\mathcal{A}}$. In the noncommutative case wi will naturally speak of this as a Noncommutative Manifold.

Let me flash a condensed version of all of the conditions without too much comments

1. **(Dimension).** There is a nonnegative integer $d$ such that the eigenvalues of $\boxed{|D|}$ grow as $\boxed{n^{\frac{1}{d}}}$

2. **(Regularity).** For any $\boxed{a \in \mathcal{A}}$ and any integer $\boxed{k}$, $\boxed{a}$ and $\boxed{[D, a]}$ belong to the domain of $\boxed{\delta^k}$, where $\boxed{\delta(a) = [|D|, a]}$.

3. **(Finiteness).** The space $\boxed{\mathcal{H}^\infty = \bigcap_k \mathrm{Dom}(D^k)}$ is a finitely generated projective left $\boxed{\mathcal{A}}$ module.

4. **(Reality).** The existence of the charge conjugation $J$ with the commutation relation fixed by the number of dimension. Moreover $\boxed{[a, Jb^* J^{-1}] = [[D, a], Jb^* J^{-1}] = 0 \ , \quad \forall\ a, b \in \mathcal{A}}$

5. **(Orientation).** For even dimensions there exists the chirality operator $\boxed{\gamma}$ with commutations rules with $\boxed{J, D}$ dependent on the number of dimensions. The operator is a representation of the volume form.

6. **(Poincaré duality).** The intersection form $\boxed{K_*(\mathcal{A}) \times K_*(\mathcal{A}) \leftarrow \mathbb{Z}}$ determined by the Fredholm index maps of the operator $\boxed{D}$ and on the $\boxed{K}$-theory $\boxed{K_*(\mathcal{A} \otimes \mathcal{A}^o)}$ is nondegenerate. This is a tough one! I am giving it without explanation, although it is very important for neutrino masses. . .

To reproduce the standard model the idea is to have a non-commutative geometry which is the product of an ordinary four dimensional manifold times a finite dimensional matrix algebra

The algebra is therefore the tensor product of continuous functions times a matrix algebra, i.e. the algebra of matrix valued functions which we represent on the Hilbert space $\boxed{\mathcal{H}}$

The vectors of the Hilbert space transform under a representation of the unitary elements of the algebra, which form the gauge group

Only fundamental and trivial representations are allowed. Luckily the standard model uses only these

Hence we look for a noncommutative geometry which is the product of the continuous commutative spectral triple introduce above by a finite noncommutative geometry represented by a noncommutative matrix algebra

$$\boxed{\mathcal{A} = C(\mathbf{R}^4) \otimes \mathcal{A}_F}$$

$$\boxed{\mathcal{H} = \mathcal{H}_c \otimes \mathcal{H}_F}$$

$$\boxed{D_0 = \gamma^\mu \partial_\mu \otimes \mathbf{I} + \gamma_5 \otimes D_F}$$

$$\boxed{\gamma = \gamma_5 \otimes \gamma_F; \;\; J = J \otimes J_F}$$

Under some rather mild assumptions (like the need to have a chiral nontrivial theory, free of U(1) anomalies) the internal geometry is described by the algebra

$$\boxed{\mathcal{A}_F = \mathbf{C} \oplus \mathbb{H} \oplus M_3(\mathbf{C})}$$

Where $\boxed{\mathbb{H}}$ is the quaternion algebra and $\boxed{M_3}$ the algebra of $\boxed{3 \times 3}$ matrices.

The gauge group of this algebra (unimodular unitary elements) is the required

$$\boxed{U(1) \otimes SU(2) \otimes SU(3)}$$

We will take however take the metric $\boxed{g^{\mu\nu} = \frac{1}{2}\{\gamma^\mu, \gamma^\nu\}}$ to be non-necessarily flat. We are thus coupling the theory to a nontrivial gravitational background with spin connection $\boxed{\omega_\mu}$.

In $\boxed{\mathcal{H}_F}$ we put (by hand) all the known fermions, replicated for three generations. It is a $\boxed{96}$ dimensional space of fermions

$$\boxed{(((2\,\text{quarks} \times 3\,\text{colours}) + 2\,\text{leptons}) \times 2\,\text{chiralities} + \text{antiparticles}) \times 3\,\text{generations}}$$

The strategy is to find the noncommutative geometry which represents the standard model, using as input almost all masses and coupling of the theory

The reason for the almost will be clear in a moment

In the basis in which $\boxed{\gamma}$ is diagonal the Dirac operator is one of my beloved $\boxed{2 \times 2}$ matrices it has the form

$$\boxed{D = \begin{pmatrix} i\gamma^\mu D_\mu + \mathbb{A} & \gamma_5 S \\ \gamma_5 S^\dagger & i\gamma^\mu D_\mu + \mathbb{A} \end{pmatrix}}$$

where

$\boxed{D_\mu = \partial_\mu + \omega_\mu}$, $\boxed{\omega_\mu}$ the spin connection.

$\boxed{\mathbb{A}}$ contains all gauge fields

$\boxed{S}$ contains all field and costants which connect the left with the right fermions: the Higgs, Yukawa couplings, mixings. . .

Here I am skipping lots of important aspects including:

- The representation of the algebra on $\boxed{\mathcal{H}}$. Highly nontrivial, reducible and with a crucial use of the charge conjugation operator J

- The role and the importance of right handed neutrinos

- The fact that for how $\boxed{\mathcal{H}}$ was biult the fermionic degrees of freedom are overcounted, and therefore there are unphysical couplings among particles with the same chirality. This is solved by the presence of $\boxed{J}$ appearing in the fermionic action $\boxed{\langle J\Psi|D\Psi\rangle}$

- The role of the renormalization flow, and the fact that all couplings are equal at the scale $\boxed{\Lambda}$

Then you "just" crack the machine, as we did for electrodynamics, using heath kernel. It is a bit hard to cranck, but in the end you get

$$
\begin{gathered}
\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \\
\frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - \\
W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - igs_w(\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - \\
W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + \\
g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w (A_\mu Z^0_\nu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \frac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
\beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \frac{2M^4}{g^2}\alpha_h - g\alpha_h M \left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\frac{1}{8}g^2 \alpha_h \left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2\right) - gMW^+_\mu W^-_\mu H - \\
\frac{1}{2}g\frac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \frac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\frac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\,(\frac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - \\
ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2 W^+_\mu W^-_\mu \left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \\
\frac{1}{8}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-\right) - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \\
\frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - \\
g^2\frac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - g^2 s_w^2 A_\mu A_\mu\phi^+\phi^- + \frac{1}{2}ig_s\,\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + \\
m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_w A_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\frac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + (\bar{u}^\lambda_j\gamma^\mu(1 - \frac{8}{3}s_w^2 + \\
\gamma^5)u^\lambda_j)\} + \frac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5){U^{lep}}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\frac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa {U^{lep}}^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda {U^{lep}}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda {U^{lep}}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda {U^{lep}}^\dagger_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda {U^{lep}}^\dagger_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \frac{g}{2}\frac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \frac{g}{2}\frac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) + \\
\frac{ig}{2}\frac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \frac{ig}{2}\frac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \frac{1}{4}\,\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa - \frac{1}{4}\,\overline{\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa} + \\
\frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \frac{g}{2}\frac{m^\lambda_u}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \frac{g}{2}\frac{m^\lambda_d}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \\
\frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j)
\end{gathered}
$$

Here the notation is as in [46], as follows.

Which believe it or not ir the full lagrangian of the standard model coupled to gravity, with all gory details, all couplings and so on, but with a couple of remarkable features:

- There was no need to input the Higgs mass among the parameters , therefore there is prediction, the mentioned $\boxed{\text{170GeV}}$

- Among the gravitational terms there are terms involving the square of the Riemann tensor

- There is a non-standard coupling of the Higgs to the gravitational background of the kind $\boxed{H^2 R}$. This point and the previous have possible cosmological interest.

I do not think the theory I sketched in the last part of the talk has already the maturity for experimental predictions, Nevertheless I still find it fascinating that a theory without so little input finds a Higgs mass relatively close to the expected value.

What is more important in my opinion is the fact that a spectral point of view, despite describing commutative and noncommutative geometries enable you to give a tentative answer to the question of this meeting.

## What is Quantum Field Theory?

## Nothing but a determinant and a trace

**Tanti auguri Manolo!!!**