Matrix Models, Emergent Spacetime and the Higgs

Fedele Lizzi

Università di Napoli Federico II

and Insitut de Ciencies del Cosmo, Universitat de Barcelona

Work very much in progress with H. Steinacker and H. Grosse

Wroclaw 2009

One of the recurrent themes of this conference is the fact that at the Planck scale the ordinary geometry may no longer be valid

And the obvious, and very much employed analogy, is the geometry of quantum phase space, often described by the Grönewold-Moyal product

This is in some sense the simplest kind of noncommutative space, characterized by a constant commutator among the coordinates

$$\boxed{[x^\mu, x^\nu] = \mathrm{i}\theta^{\mu\nu}}$$

This is not the place to describe successes and problems of this noncommutative space, but I think we all agree that its implementation, with a constant θ is too naive.

In this talk I will briefly describe an approach to noncommutative spaces based on matrix models, mainly work of H. Steinacker and collaborators (P. Aschieri, T. Grammatikopoulos, H. Grosse, D. Klammer, FL, M. Wohlgenannt, G. Zoupanos . . .)

For a similar approach see the work of H.S. Yang

The general programme is to describe noncommutative space-time as a matrix model with a simple action, inserting fermions, and have gravity, and other physical characteristic, emerge as fluctuations around a semiclassical vacuum

I will be very impressionistic and will not give details. Some times it would take too long to give the details, sometimes I do not know the details, sometimes the details have to be still worked out.

In this talk I will briefly describe an approach to noncommutative spaces based on matrix models, mainly work of H. Steinacker and collaborators (P. Aschieri, T. Grammatikopoulos, H. Grosse, D. Klammer, FL, M. Wohlgenannt, G. Zoupanos . . .)

For a similar approach see the work of H.S. Yang

The general programme is to describe noncommutative space-time as a matrix model with a simple action, inserting fermions, and have gravity, and other physical characteristic, emerge as fluctuations around a semiclassical vacuum

I will be very impressionistic and will not give details. Some times it would take too long to give the details, sometimes I do not know the details, sometimes the details have to be still worked out. I hope you will let me "get away with murder", or at lest with "serious bodily injury".

The rationale behind this approach is quite simple and can be very heuristically (and therefore incorrectly) be stated as follows

- A noncommutative geometry is described by a noncommutative algebra, deformation of the commutative algebra of function on some space

- Any noncommutative ($\boxed{C^*}$)-algebra is represented as operators on some Hilbert space

- Operators on an Hilbert space are just infinite matrices

Consider a $\boxed{U(1)}$ gauge theory in a space described by the Moyal $\boxed{\star}$ product

The theory is noncommutative (also in the $\boxed{U(1)}$ case), due to the noncommutativity of the product

$$\boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}[A_\mu, A_\nu]_\star}$$

Consider as usual the action to be the square of the curvature

$$\boxed{S = -\frac{1}{4}\int \mathrm{d}x F_{\mu\nu} \star F^{\mu\nu}}$$

The theory is invariant for $\boxed{F \longrightarrow U \star F \star U^\dagger}$ for $\boxed{U \star U^\dagger = 1}$

Considering $\boxed{A_\mu = A_\mu^\alpha \lambda^\alpha}$ for $\boxed{\lambda^\alpha}$ generators of $\boxed{U(n)}$ allows for noncommutative Yang-Mills theories

For the Moyal product $\boxed{\partial_\mu(f) = \mathrm{i}\theta^{-1}_{\mu\nu}[x^\nu, f]}$

If one defines (Madore, Schraml, Schupp, Wess) the covariant coordinates $\boxed{X^\mu = x^\mu + \theta^{\mu\nu} A_\nu}$ and

$$\boxed{D_\mu f = \mathrm{i}\theta^{-1}_{\mu\nu}[X^\mu, f]_\star = \partial_\mu f - \mathrm{i}[f, A_\mu]_\star}$$

we have

$$\boxed{F^{\mu\nu} = [D^\mu, D^\nu]_\star = [X^\mu, X^\nu]_\star + \theta^{\mu\nu}}$$

And the constant can be reabsorbed by a field redefinition. The action is the square of this quantity, integrated over spacetime

The objects we have defined are elements of a noncommutative algebra and we can always represent them as operators on a Hilbert space, in this case the integral becomes a trace and this suggests the use of the matrix action

$$\boxed{S = -\frac{1}{4g}\,\mathrm{Tr}\,[X^\mu, X^\nu][X^{\mu'}, X^{\nu'}]g_{\mu\mu'}g_{\nu\nu'}}$$

Where the $\boxed{X}$'s are operators (matrices) and the metric $\boxed{g_{\mu\mu'}}$ is the flat Minkowski (or Euclidean) metric

The fascinating characteristic of this action is that gravity emerges from it.

The equations of motion are

$$\boxed{[X^\mu, [X^\nu, X^{\mu'}]]g_{\mu\mu'} = 0}$$

A possible vacuum (the $\boxed{U(1)}$ Moyal vacuum) given by a set of matrices $\boxed{X_0}$ such that $\boxed{[X_0^\mu, X_0^\nu] = \mathrm{i}\theta^{\mu\nu}}$ con $\boxed{\theta}$ constant

This is some sort of semiclassical vacuum and we can consider $\boxed{f(X_0)}$ as deformation of functions on a Moyal deformed space

This is the vacuum described earlier.

Now let fluctuate these "coordinates" and consider $\boxed{X^\mu = X_0^\mu + A^\mu}$ so that $\boxed{[X^\mu, X^\nu] = \mathrm{i}\theta(X)}$ and we are considering a nonconstant noncommutativity

Gravity emerges as nontrivial curvature considering for example the coupling with a scalar field $\boxed{\Sigma}$. The (free action) is

$$\boxed{\mathrm{Tr}\,[X^\mu, \Sigma][X^\nu, \Sigma]g_{\mu\nu} \sim \int \mathrm{d}x (D_{\mu'}\Sigma)(D_{\nu'}\Sigma)\theta^{\mu\mu'}\theta^{\nu\nu'}g_{\mu\nu} = \int \mathrm{d}x (D_\mu\Sigma)(D_\nu\Sigma)G^{\mu\nu}}$$

where we have defined the new, (non flat) metric $\boxed{G^{\mu\nu}(x) = \theta^{\mu\mu'}\theta^{\nu\nu'}g_{\mu'\nu'}}$

Effectively a curved background has emerged from noncommutativity. The gravitational action is recovered as effective action at one loop

An alternative vacuum, still solution of the equations of motion, is

$$\boxed{Y_0^\mu = X_0^\mu \otimes \mathbb{1}_n}$$

Consider as before the fluctuations

$$\boxed{Y = Y_0 + A_0 + A_\alpha \lambda_\alpha}$$

where in the fluctuations we have separated the traceless generators of $\boxed{SU(n)}$ from the trace part ($\boxed{A_0}$)

The $\boxed{U(1)}$ trace part of the fluctuation gives rise to the gravitational coupling, while the remaining $\boxed{A_\alpha}$ describe a $\boxed{SU(n)}$ gauge theory

How to get closer to the standard model?

The plan is to find a matrix model with some coordinates (a vacuum) and an action which reproduces, as close as possible, the standard model with breaking $\boxed{SU(3) \times SU(2) \times U(1) \to SU(3) \times U(1)}$

We should not be shy of making as many assumptions as are needed. The game is not to find the standard model, but rather to find a noncommutative geometry which "fits" it

We have already managed to find a $\boxed{SU(n)}$ theory. We need two more stages, first a modification of the model to allow $\boxed{SU(3) \times SU(2) \times U(1)}$, and then a symmetry breaking mechanism

We also have to put fermions (in the right representation)

What we are proposing is in some sense a fully noncommutative version of the Connes-Lott and Chamseddine-Connes-Marcolli models

In their case the geometry is “almost commutative”, the product of ordinary spacetime by a finite dimensional matrix, and the action is either the square of the curvature (as basically is our case) or the spectral action

The model I will present is incomplete and can be considered as a first approximation. Remarkably however some key characteristics of the standard model emerge naturally, which makes us confident that a fully viable (and predictive) model is within reach

The first stage can be accomplished by considering the following vacuum for which we add another coordinate, for which I will use the index $\boxed{\Phi}$ to differentiate it form the usual coordinates

$$\boxed{X^\mu = X_0^\mu \otimes \mathbb{1}_n \quad , \quad X^\Phi = \begin{pmatrix} \alpha_1 \mathbb{1}_{n_1} & \\ & \alpha_2 \mathbb{1}_{n_2} \end{pmatrix}}$$

with $\boxed{n_1 + n_2 = n}$ and $\boxed{\alpha_i \in \mathbf{R}}$

The equations of motion are still satisfied but the gauge group is reduced to $\boxed{SU(n_1) \times SU(n_2) \times U(1)}$ because one $\boxed{U(1)}$ is the one giving rise to gravity

The contribution to the action is a term of the kind

$$\boxed{F^{\mu\Phi} = [\bar{X}^{\mu} + A^{\mu}, A^{\Phi}] + [A^{\mu}, X^{\Phi}]}$$

$$\boxed{[A^{\mu}, X^{\Phi}] = \begin{pmatrix} 0 & (\alpha_2 - \alpha_1)A^{\mu}_{12} \\ (\alpha_1 - \alpha_2)A^{\mu}_{21} & 0 \end{pmatrix}}$$

where we consider the block form of A^{μ}

$$\boxed{A^{\mu} = \begin{pmatrix} A^{\mu}_{11} & A^{\mu}_{12} \\ A^{\mu}_{21} & A^{\mu}_{22} \end{pmatrix}}$$

If the differences $\boxed{\alpha_1 - \alpha_2}$ is large, all non diagonal blocks of $\boxed{A^{\mu}}$ acquire large masses decoupling

A matrix model with this sort of gauge group can also be obtained by considering the $\boxed{\alpha_i}$ as the effective remnant of a pair of fuzzy sphere of slight different order. Clearly the mechanism works also for more than two groups

Hence there are different mechanisms that may lead to a vacuum for which at a certain scale the vacuum is (I have not written one or more $\boxed{X^\Phi}$)

$$\boxed{X^\mu = \begin{pmatrix} X_0^\mu \otimes \mathbb{1}_2 & & & \\ & X_0^\mu & & \\ & & X_0^\mu & \\ & & & X_0^\mu \otimes \mathbb{1}_3 \end{pmatrix}}$$

The generators of the gauge group are for a $\boxed{SU(2)}$ and $\boxed{SU(3)}$ respectively are

$$\boxed{W^\mu = \begin{pmatrix} w & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}}$$

$$\boxed{G^\mu = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & g^\mu \end{pmatrix}}$$

Where $\boxed{w, g}$ are in the adjoint representation

The fluctuations of the coordinates are given by

$$\boxed{A^\mu = \begin{pmatrix} A_2^\mu & & & \\ & a^\mu & & \\ & & a'^\mu & \\ & & & A_3^\mu \end{pmatrix}}$$

The gauge group is $\boxed{SU(3) \times SU(2) \times U(1)^3}$, we have the standard model group plus some extra $\boxed{U(1)}$'s, but we are getting closer

We need some fermions which transform properly under the gauge group. In general for matrix models the fermions transform under the adjoint action of the group

The action for fermions is

$$\boxed{S_F = \mathrm{Tr}\,\bar{\Psi}\gamma_a[X^a, \Psi] \sim \int \mathrm{d}x \bar{\Psi}\gamma_a D^a \Psi}$$

We can accommodate all known fermions (except right handed $\boxed{\nu}$) in the following upper diagonal matrix

$$\boxed{\Psi = \begin{pmatrix} 0_{2\times 2} & 0_{2\times 1} & L_L & Q_L \\ 0_{1\times 2} & 0_{1\times 1} & e_R & d_R \\ 0_{1\times 2} & 0_{1\times 1} & 0_{1\times 1} & u_R \\ 0_{3\times 2} & 0_{3\times 1} & 0_{3\times 1} & 0_{3\times 3} \end{pmatrix}}$$

With $\boxed{L_L = \begin{pmatrix} e \\ \nu \end{pmatrix}}$ is the fermion doublet and the quarks are arranged horizontally (colour acts on the right for them.)

Right handed neutrinos (and in general particles with Majorana mass) can be put on the diagonal and there is room for an exotic left handed doublet

The generator of Hypercharge is

$$\boxed{Y = \begin{pmatrix} 0_{2\times 2} & & \\ & -\sigma_3 & \\ & & -\frac{1}{3}\mathbb{1}_{3\times 3} \end{pmatrix} + \frac{1}{7}\mathbb{1}_7}$$

And the Baryon number is

$$\boxed{B = \begin{pmatrix} 0 & & \\ & 0 & \\ & & -\frac{1}{3}\mathbb{1}_{3\times 3} \end{pmatrix} + \frac{1}{7}\mathbb{1}_7}$$

The constants are there to make the generators traceless

Let us consider another the extra coordinate:

$$\boxed{X^{\varphi} = \begin{pmatrix} 0 & \varphi & 0 & 0 \\ \varphi^{\dagger} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}$$

where the vacuum value is $\boxed{\varphi = \begin{pmatrix} 0 \\ v \end{pmatrix}}$

Since $\boxed{[X^{\mu}, X^{\varphi}] = 0}$ the extra coordinate is still solution of the equation of motion, with $\boxed{\theta^{\mu\varphi} = 0}$ which implies that in the semi-classical limit $\boxed{G^{\mu\varphi} = 0}$

The extra dimensions are not dynamical

This extra coordinate does not commute with the generators of the weak $\boxed{SU(2)}$ and with the hypercharge $\boxed{Y}$, but it does commute with the electric charge

$$\boxed{Q = w_3 + \frac{Y}{2} = \frac{1}{2}\begin{pmatrix} \sigma_3 & & \\ & -\sigma_3 & \\ & & -\frac{1}{3}\mathbb{1}_{3\times 3} \end{pmatrix} + \frac{1}{7}\mathbb{1}_7}$$

Identifying $\boxed{\gamma^\varphi = \gamma^5}$, so to have the correct Clifford algebra in five dimensions, the fermionic part of the action reproduces the form of the Yukawa couplings and mass terms of the standard model

$$\boxed{S_Y = \operatorname{Tr} \bar\Psi \gamma_\phi [X^\varphi, \Psi] = \bar e_r \varphi^\dagger L_L + \bar Q_L \varphi u_r + \bar d_r \varphi^\dagger Q_L}$$

The matrix model captures some characteristics of the standard model

The list of drawbacks and physically unrealistic features of this simple model is probably too long to be mentioned fully

It comprises the fact that at the bare level the Yukawa couplings are the same for leptons and quarks (but this may change under renormalization), that the unbroken gauge group is still too large, and the fact that the three generations are treated in the same way, there are no generations

The extra coordinates structure will have to be noncommutative itself and several improvements are possible, and currently under investigation

Conclusions

I hope I managed to convince you that some matrix models could have a semiclassical limit describing the standard model and gravity as an emergent phenomenon

At the present we can manage to reproduce some key features of it in a rather simple (and naive) model with a couple of extra coordinates

As the extra dimension get more structure the model becomes more realistic, and I am convinced that in a near future it will be possible to make definite predictions, in fact some cosmological predictions have already appeared

The analogy with Connes' approach is evident, although the action is different, probably however some sort of spectral action could be built for these models, and the full power of the mathematics on noncommutative geometry can be applied